Master's Program in Medical Physics

4. LASER Resonators

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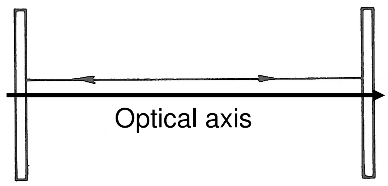
Ideal Resonator

Highly reflecting, flat, parallel mirrors



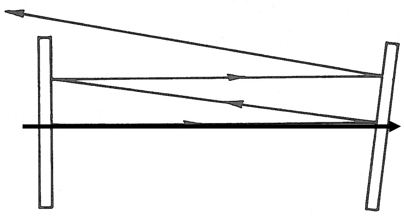
Assumptions

1. Uniform field in any plane perpendicular to the optical axis

2. Laser Medium is passive
   → No influence on electromagnetic modes

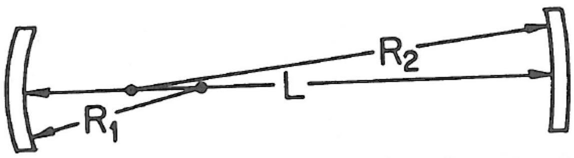
☑ uniform distribution of
- gain coefficient
- refractive index

Realistic Situation



Problem: Slight misarrangement of mirror(s)

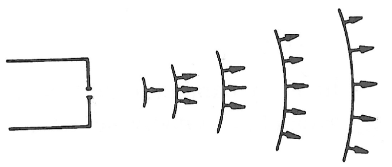
→ more realistic resonator geometries



Geometrical Optics

Propagation of light: Rays

- Each point of a wave, vector representation
- Direction:
  - normal to wave front
  - propagation direction of energy flow
- Length: no physical significance

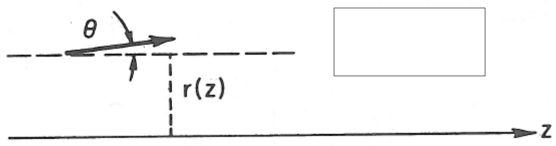


☒ Polarization of light ignored

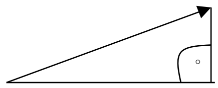
Ray Propagation

Light propagation: 1 single direction = along optical axis → **Paraxial rays**

1. Displacement r(z)



2. Slope r'(z)



dr

dz

$$r'(z) = \frac{dr}{dz} = \tan\theta$$

Matrix notation

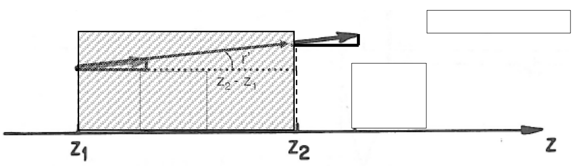
$$\begin{bmatrix} r \\ r' \end{bmatrix}$$

2x1 matrix

(column vector)

Ray Propagation: In Vacuum

$$r'(z) = \frac{dr}{dz} = \tan\theta$$



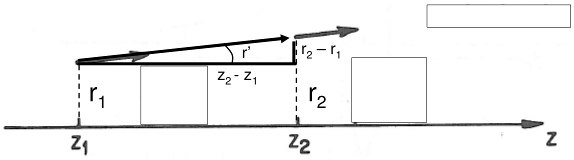
$$r'(z_2) = r'(z_1)$$

$$r(z_2) = \underbrace{r(z_1)} + \underbrace{(z_2 - z_1) \cdot r'(z_1)}$$

Matrix notation

$$\begin{bmatrix} r(z_2) \\ r'(z_2) \end{bmatrix} = \begin{bmatrix} 1 & z_2 - z_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} r(z_1) \\ r'(z_1) \end{bmatrix} \Leftrightarrow \begin{bmatrix} r_f \\ r'_f \end{bmatrix} = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} r_i \\ r'_i \end{bmatrix}$$

Sign convention: Slope

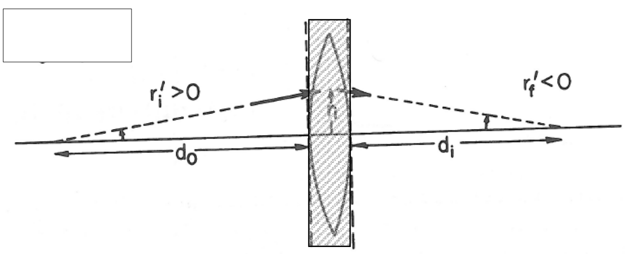


$$r'(z) = \frac{dr}{dz} = \tan\theta$$

$r_2 > r_1$ $\qquad r' > 0$

$r_2 < r_1$ $\qquad r' < 0$

Ray Propagation: Thin Lens



Thin lens equation

$$\frac{1}{d_0} + \frac{1}{d_i} = \frac{1}{f}$$

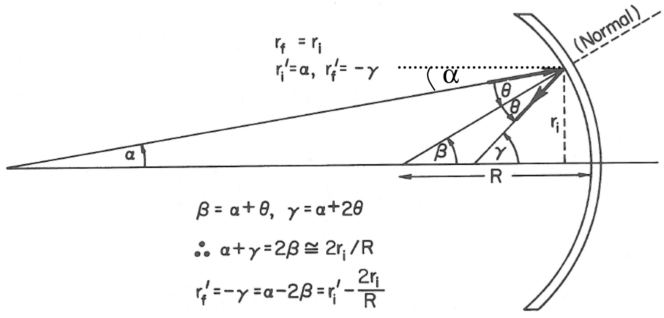
Matrix notation

$$r_f = r_i$$

$$r'_f = r'_i - \frac{r_i}{f}$$

$$\begin{bmatrix} r_f \\ r'_f \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \begin{bmatrix} r_i \\ r'_i \end{bmatrix}$$

Ray propagation: Spherical Mirror



$$r_f = r_i$$

$$r'_f = r'_i - \frac{2r_i}{R}$$

Matrix notation

$$\begin{bmatrix} r_f \\ r'_f \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2/R & 1 \end{bmatrix} \begin{bmatrix} r_i \\ r'_i \end{bmatrix}$$

Mirror: Concave ⇔ Convex

biconvex concave-convex biconcave convex-concave

plano-convex plano-concave

Sign convention

⇩

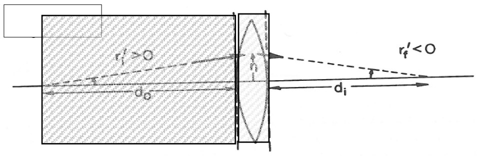
R > 0, f > 0 (convex) R < 0, f < 0 (concave)

Multiple Optical Elements

Ray matrix

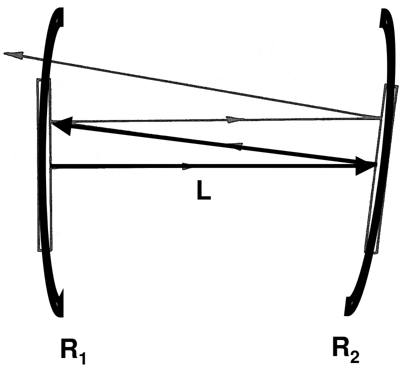
$$\begin{bmatrix} r_f \\ r'_f \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} r_i \\ r'_i \end{bmatrix}$$

Open path (d) + Thin lens (f)



$$\begin{bmatrix} r_f \\ r'_f \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} r_i \\ r'_i \end{bmatrix} = \begin{bmatrix} 1 & d \\ -1/f & 1-d/f \end{bmatrix} \begin{bmatrix} r_i \\ r'_i \end{bmatrix}$$

Resonator Stability I



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2/R_1 & 1 \end{bmatrix} \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2/R_2 & 1 \end{bmatrix} \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix}$$

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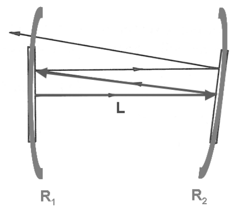
Resonator Stability II



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1-\frac{2L}{R_2} & 2L-\frac{2L^2}{R_2} \\ \frac{4L}{R_1R_2}-\frac{2}{R_1}-\frac{2}{R_2} & 1-\frac{2L}{R_2}-\frac{4L}{R_1}+\frac{4L^2}{R_1R_2} \end{bmatrix}$$

<u>After N round trips</u>

$$\begin{bmatrix} r_N \\ r_N' \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^N \begin{bmatrix} r_i \\ r_i' \end{bmatrix}$$

Determinant: $AD - BC = 1$

Definition: $\cos\theta = \frac{1}{2}(A + D)$

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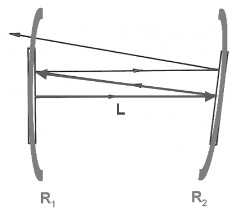
Resonator Stability III



$$\cos\theta = \frac{1}{2}(A + D)$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}^N = \frac{1}{\sin\theta} \begin{bmatrix} A\sin(N\theta) - \sin((N-1)\theta) & B\sin(N\theta) \\ C\sin(N\theta) & D\sin(N\theta) - \sin((N-1)\theta) \end{bmatrix}$$

Ray convergence criterion: r_n , r_n' finite $\Rightarrow$ θ real

$$|\cos\theta| \le 1$$

$$0 \le 1 - \frac{L}{R_1} - \frac{L}{R_2} + \frac{L^2}{R_1R_2} \le 1$$

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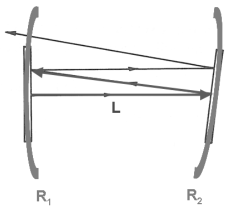
Stability criterion



$$0 \le 1 - \frac{L}{R_1} - \frac{L}{R_2} + \frac{L^2}{R_1 R_2} \le 1$$

LASER literature: $0 \le g_1 g_2 \le 1$

g parameters of resonator: $g_1 = 1 - \frac{L}{R_1}$ $\quad g_2 = 1 - \frac{L}{R_2}$

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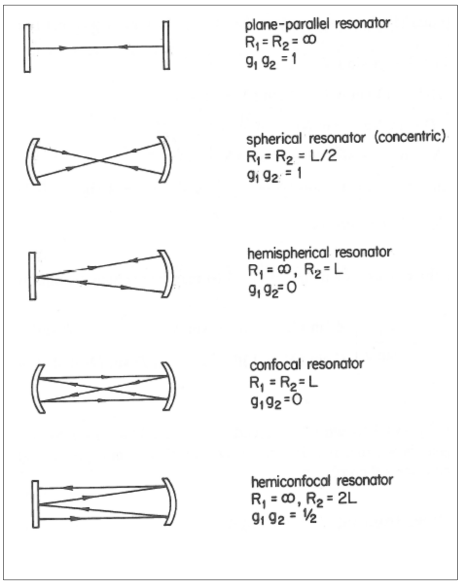
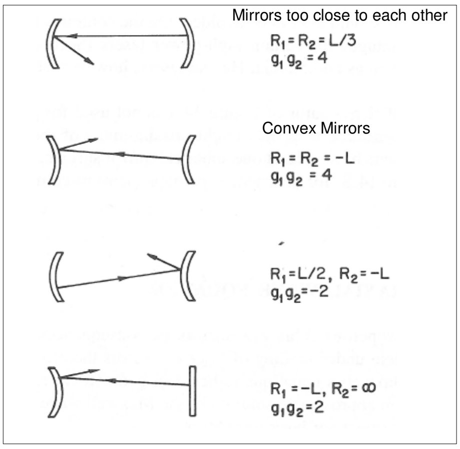
Resonator Examples



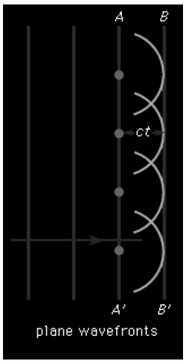


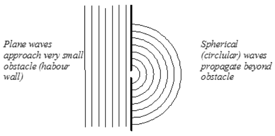
$$0 \le \underbrace{(1 - \frac{L}{R_1})}_{g_1} \cdot \underbrace{(1 - \frac{L}{R_2})}_{g_2} \le 1$$

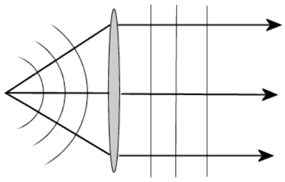
Monochromatic Fields

Plane Wave

$$\varepsilon(\vec{r}) = \varepsilon_0 \, e^{-i\vec{k}\cdot\vec{r}}$$

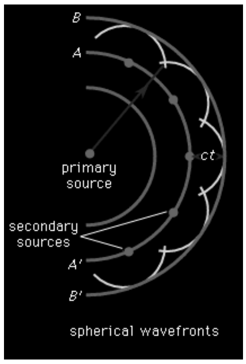






Spherical Wave

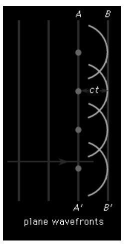
$$\varepsilon(\vec{r}) = \frac{A}{r} \, e^{-ik\cdot r}$$



Wave Description of a LASER Beam

Plane Wave

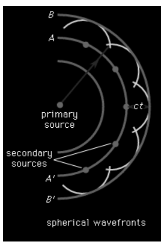
$$\varepsilon(\vec{r}) = \varepsilon_0 \, e^{-i\vec{k}\cdot\vec{r}}$$



infinite
cross-sectional
area

Spherical Wave

$$\varepsilon(\vec{r}) = \frac{A}{r} \, e^{-ik\cdot r}$$



not
uni-directional

More Realistic

$$\varepsilon(\vec{r}) = \frac{A}{R} \, e^{ik\frac{(x^2+y^2)}{2R}} \, e^{ikR}$$

R=z (in the vicinity of the optical axis)

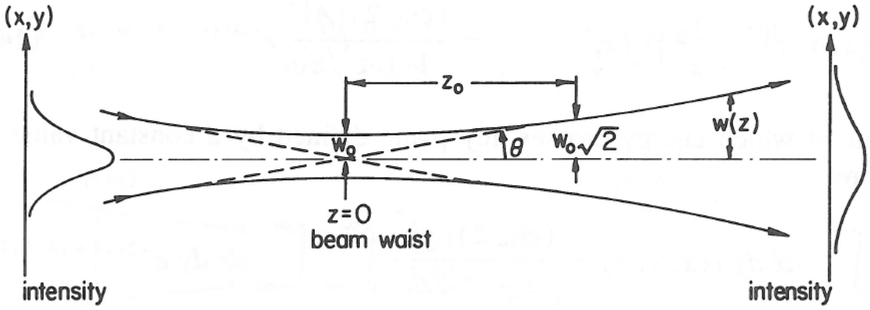
↓

$$\varepsilon(\vec{r}) = \varepsilon_0(\vec{r}) \, e^{ikz}$$

- Nearly uni-directionality of a plane wave
- Without having an infinite beam cross section

Beam: Waist

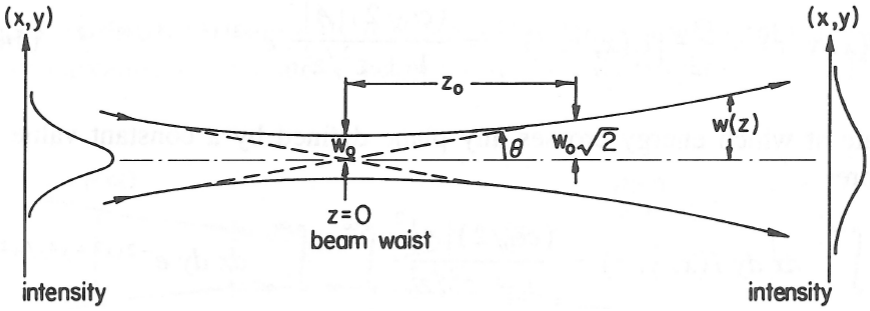
w(z): spot size varies with propagation distance



$$z_0 = w_0^2 \frac{k}{2} = \frac{w_0^2 \pi}{\lambda} \qquad w(z_0) = w_0\sqrt{2}$$

The Rayleigh range is a measure of the length of waist region

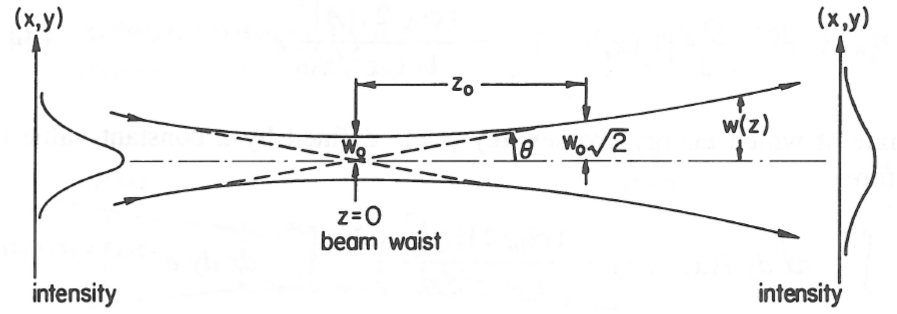
Beam: Divergence Angle



For $z >> z_0$:

$$\theta \approx \frac{w(z)}{z} \approx \frac{w_0}{z} = \frac{\lambda}{w_0 \pi} \qquad w(z) = w_0\sqrt{1+\frac{z^2}{z_0^2}}$$

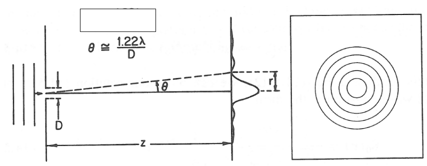
Beam: Divergence Angle



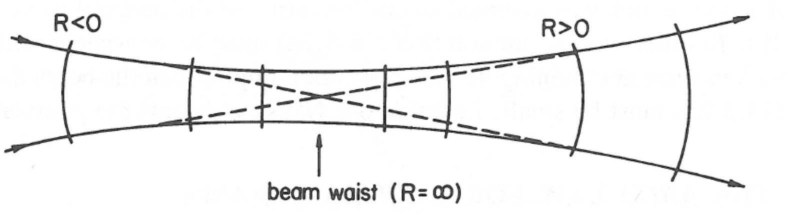
For z >> z_0 :

$$\theta \approx \frac{w(z)}{z} \approx \frac{w_0}{z} = \frac{\lambda}{w_0 \pi}$$

Analogy



Beam: Radius of Curvature



$$R(z) = z + \frac{z_0^2}{z}$$

For z >> z_0 : $R(z) \approx z$

Surfaces of constant phase are spheres centred on the beam waist

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Gaussian Beams: Summary

$$E(\mathbf{r}) = \mathcal{E}(\mathbf{r})\, e^{-i\omega t} \qquad \text{(electric field)}$$

$$\mathcal{E}(\mathbf{r}) = A \frac{w_0}{w(z)} e^{i[kz - \tan^{-1}(z/z_0)]} e^{ik(x^2+y^2)/2R(z)} e^{-(x^2+y^2)/w^2(z)}$$

$$I(\mathbf{r}) = \frac{c\epsilon_0}{2} |A|^2 \left(\frac{w_0}{w(z)}\right)^2 e^{-2(x^2+y^2)/w^2(z)} \qquad \text{(intensity)}$$

$$w(z) = w_0 \sqrt{1 + \frac{z^2}{z_0^2}} \qquad \text{(spot size)}$$

$$R(z) = z + \frac{z_0^2}{z} \qquad \text{(radius of curvature)}$$

$$z_0 = \pi w_0^2/\lambda \qquad \text{(Rayleigh range)}$$

$$\theta = \lambda/\pi w_0 \qquad \text{(divergence angle)}$$

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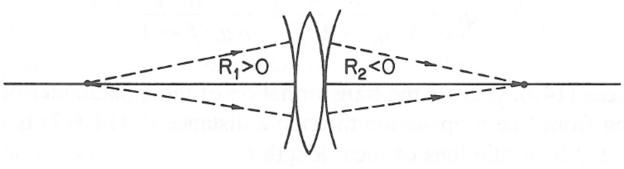
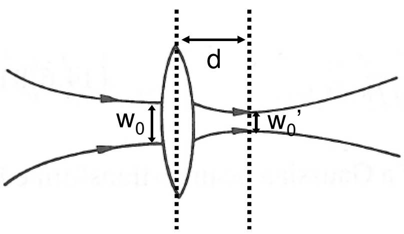
Gaussian Beam: Thin Lense



$d_o = R_1,\; d_i = -R_2,\; \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$

$$\frac{1}{R_2} = \frac{1}{R_1} - \frac{1}{f}$$



$$d = f \Big/ \left(1 + \frac{f^2}{z_0^2}\right)$$

$$w_0' = \frac{f\lambda}{w_0 \pi} \frac{1}{\sqrt{1 + f^2/z_0^2}}$$

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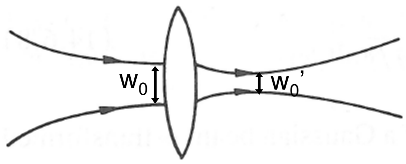
Example: He-Ne LASER



$$w_0' = \frac{f\lambda}{w_0 \pi} \frac{1}{\sqrt{1+f^2/z_0^2}}$$

Spot size at the waist: w_0 = 1 mm

LASER wave length: λ = 632.8 nm

For f << z_0 :

$w_0' \approx 2 \cdot 10^{-4}$ f

A Gaussian Beam can be focused to a very small spot !

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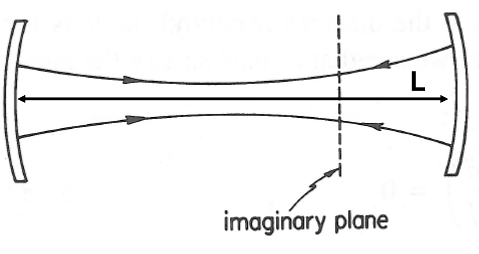
Resonator Mode: Definition



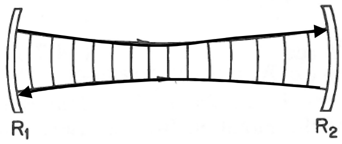
Time: T = L / c

Traverse Intensity Profile:

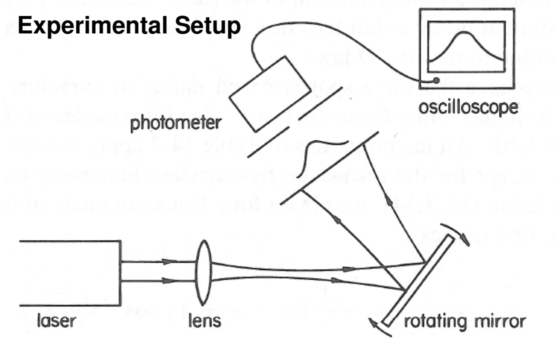
Steady profile not changing with time

Resonator Mode: Steady spatial pattern of the field inside a resonator

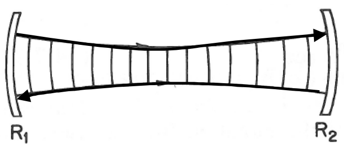
Resonator Mode: Measurement

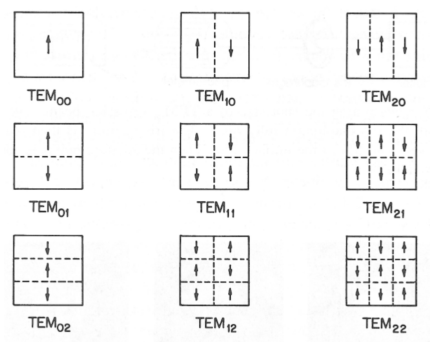


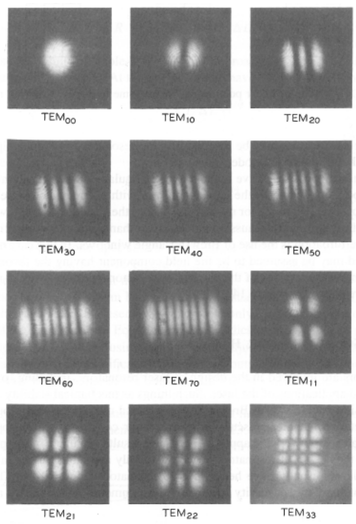
Resonator Mode Generation:
Interference of
right-going and **left-going** beam
between the mirrors



Resonator Mode: Examples



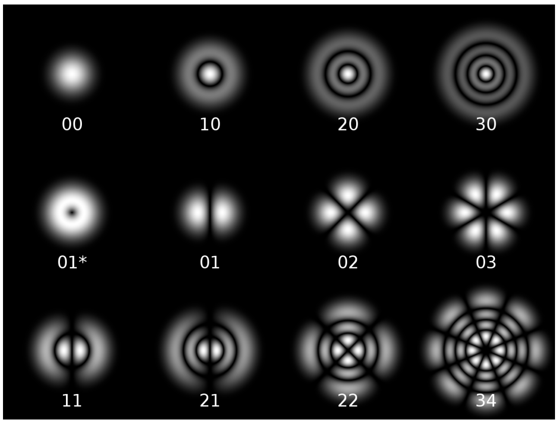




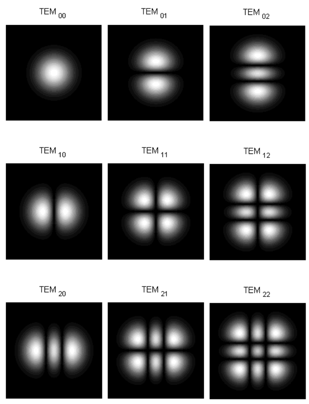
Example: He-Ne-LASER

Resonator Modes: Geometries

Cylindrical



Rectangular



Next Lecture

5. LASER – Tissue Interactions I

Matter acting on Light

- Reflection
- Refraction
- Absorption
- Scattering

Light acting on Matter

- Photochemical Interaction
- Thermal Interaction
- Photoablation
- Plasma-Induced Ablation
- Photodisruption