Master‘s Program in Medical Physics

Biomedical Engineering

Diffusion Weighted Imaging in MRI

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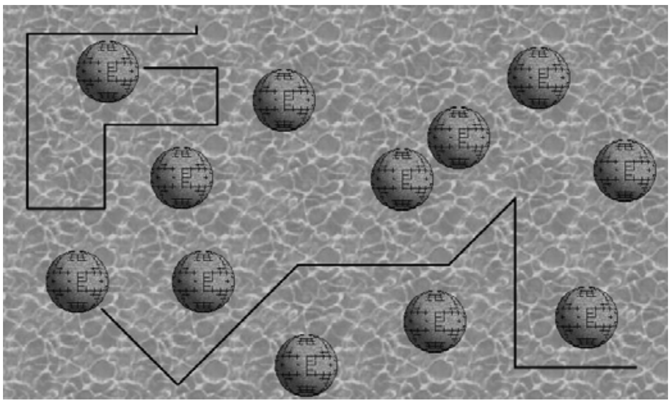
Free Diffusion



- mean diffusion length $\langle x^2 \rangle = 6Dt$

  (D = diffusion coefficient, t = time)

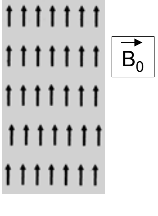
MRI

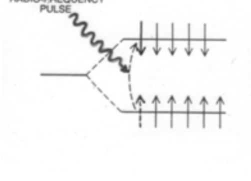
MRI Background

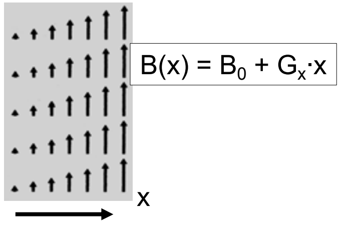
Gradients

Magnetic Resonance Imaging: Principle

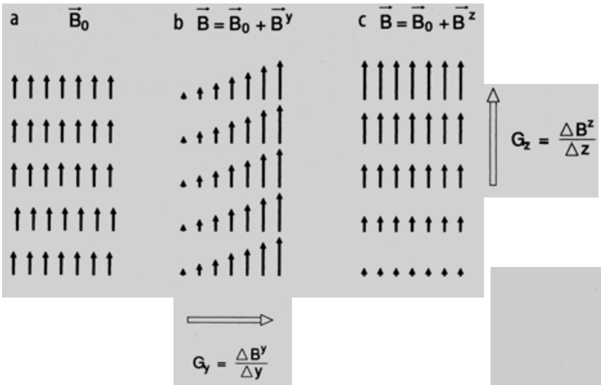
interaction of spins with static magnetic field $B_0 \rightarrow M_0$	interaction of protons with resonant magnetic radiofrequency field $B_1 \rightarrow$ FID signal	linear magnetic field gradient across the object G → spatial encoding
$\omega = \gamma \cdot B_0$	$\Delta E = \hbar\omega$	$\omega(x) = \gamma \cdot B(x)$

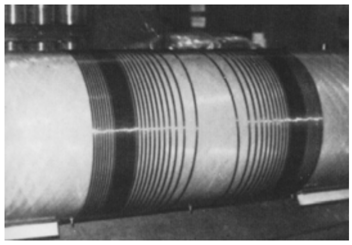


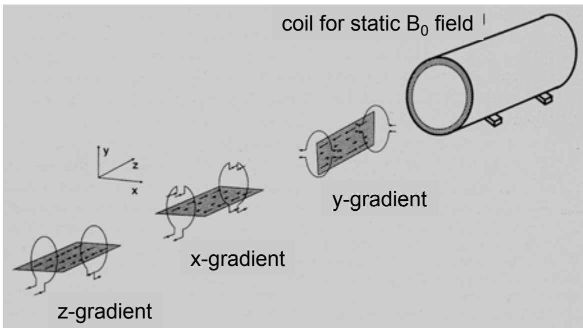




Gradient Coil: Principle I







typical value for G:
1 - 25 (40) mT/m

example:
x = 30 cm, B_0 = 1 T, G_x = 10 mT/m
B(x=30cm) = 1.0003 T

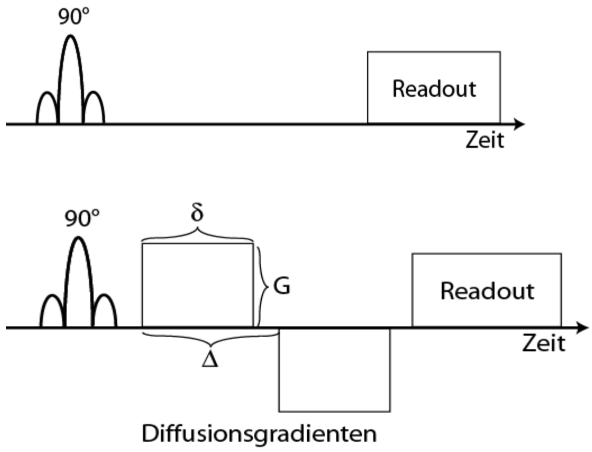
Diffusion

Diffusion Weighted Imaging (DWI)

Principle of DWI I

without diffusion weighting:

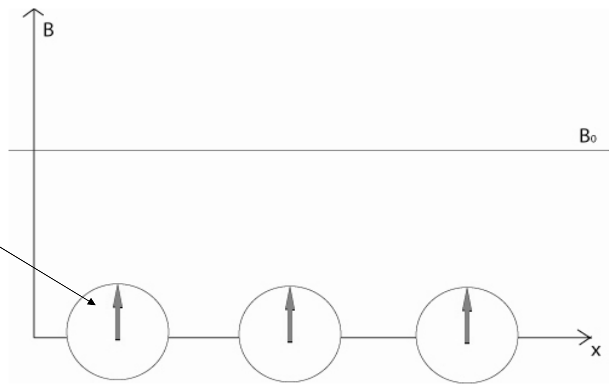
with diffusion weighting:



Principle of DWI II

after 90° pulse

position of the spin in the rotating reference frame



constant phase in the rotating reference frame

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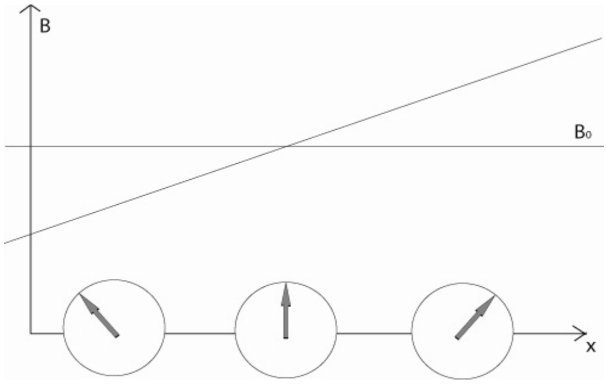
Principle of DWI III

without diffusion:



1. gradient → relative phase shift in the rotating frame

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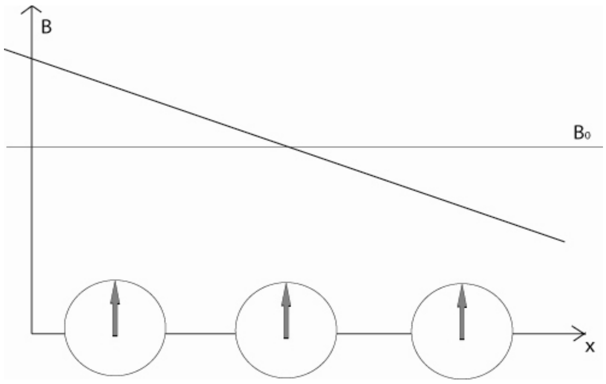
Principle of DWI IV

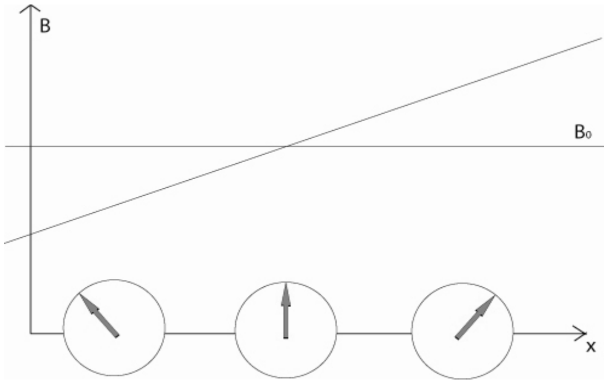
without diffusion:



2. gradient → rephasing of the phase → gradient echo

Principle of DWI V

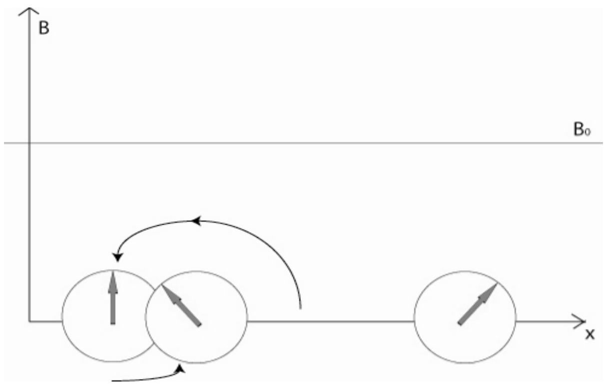
with diffusion:



1. gradient → relative phase shift in the rotating frame

Principle of DWI VI

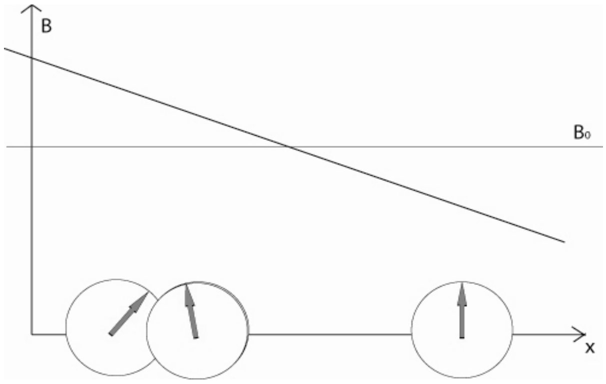
with diffusion:



diffusion during the gradients → spins on a different position

Principle of DWI VII

with diffusion:



2. gradient → spins not completely rephased → signal reduction

Bloch equation with diffusion

$$\frac{\partial \vec{M}}{\partial t} = \gamma \vec{M} \times \vec{B} + D\Delta\vec{M}$$ (without relaxation)

mit $\vec{B}(\vec{r},t) = (0,0,\vec{G}(t)\vec{r}+B_0)^T$ und $M_+ = M_x + iM_y$

$$\frac{\partial M_+}{\partial t} = -i\gamma \vec{G}(t)\vec{r}M_+ + D\Delta M_+$$ (isotropic and homogenous)

Bloch equation with diffusion 1

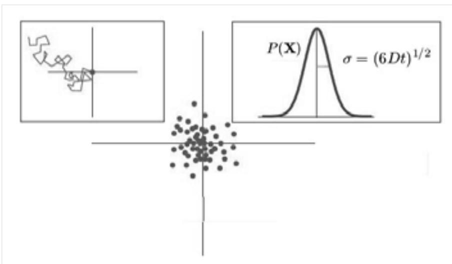
$$M_+ = A(t) e^{-i\vec{r}\gamma \underbrace{\int_0^t \vec{G}(t)dt}_{\vec{k}(t)}}$$

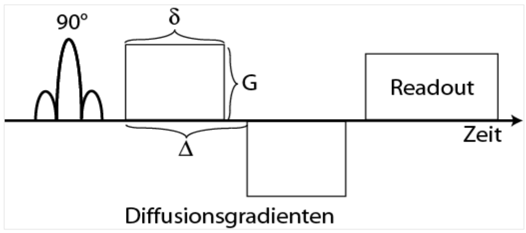
$$A(t) = A(0) e^{-D \int_0^{TE} \vec{k}^2(t) dt} \qquad \text{mit } b = \int_0^{TE} \vec{k}^2(t) dt$$

$$S = S_0 e^{-bD} \qquad \Rightarrow \ln\left(\frac{S}{S_0}\right) = -bD$$

Dependence of the Signal Decay

Free diffusion





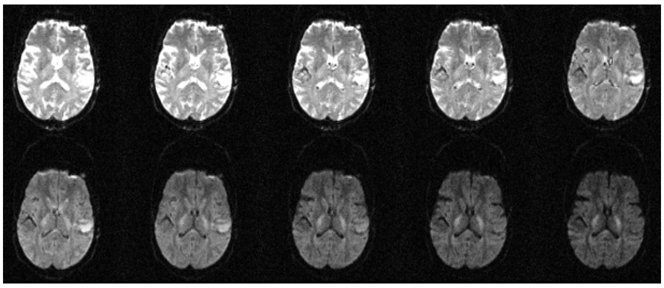
Signal decay: $S = S_0 e^{-bD}$

Strength of the diffusion weighting:

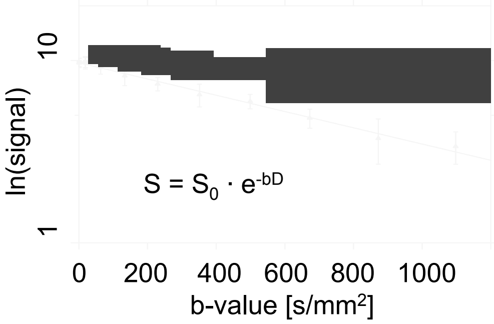
$$b = \gamma^2 G^2 \delta^2 \left(\Delta - \delta/3\right)$$

• b-value can be calculated from the sequence
• two DWIs are necessary to calculate D

Diffusion Measurement: Calculation of D



b-values = 0.5 – 1100 s/mm²

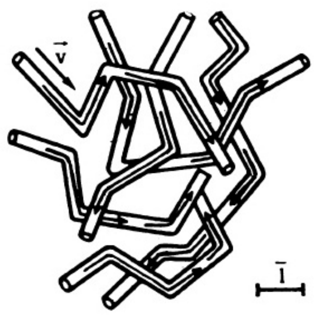


white matter
$D \sim 0.6 \cdot 10^{-3}$ mm²/s

water
$D \sim 2.4 \cdot 10^{-3}$ mm²/s

Measurement of D in biological tissue is called ADC

Intra voxel incoherent motion (IVIM)



IVIM-Theory:

$$\frac{S}{S_0} = (1-f)\,\mathrm{e}^{-bD} + f e^{-b(D^*+D)}$$

f = perfusion fraction
D = diffusion coefficient
D* = pseudo diffusion coefficient

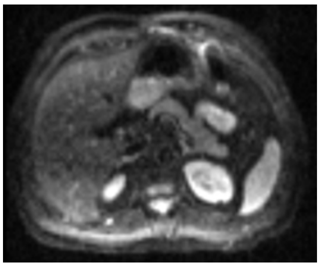
- Movement of blood spins can be interpreted as a diffusion process

→ Biexponential signal decay

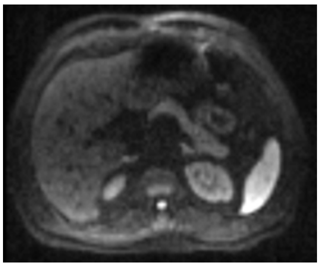
Le Bihan, Radiology, 1988

Dependence of the ADC to the applied b-value

b = 0 s/mm²

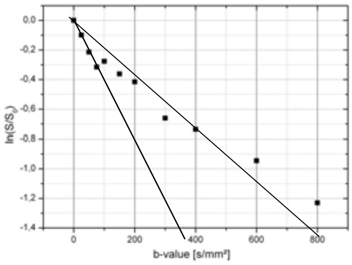


b = 400 s/mm²



ADC = 1650 µm²/s

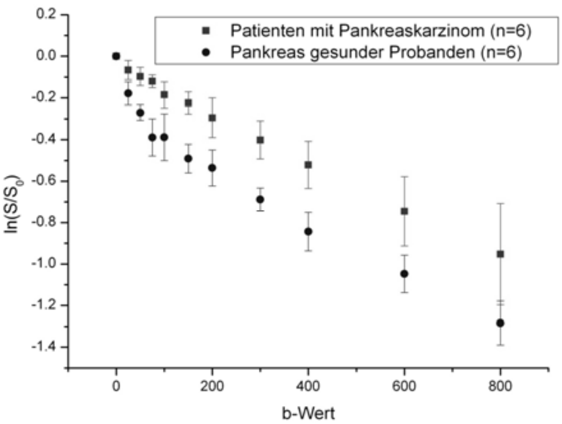
ADC = 4260 µm²/s



ADC-map [µm²/s]

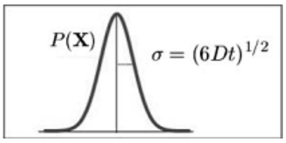
- Varying ADC values
- Signal decay is not monoexponential

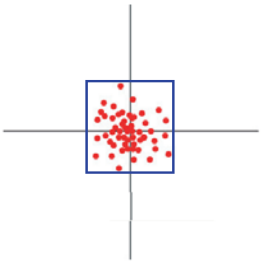
Differentiation between pancreas carcinoma and healthy pancreas



- Almost monoexponential signal decay in pancreas carcinoma
- At least biexponential in healthy pancreatic tissue
- No overlap between the two groups at low b-values

Restricted diffusion and q-space imaging



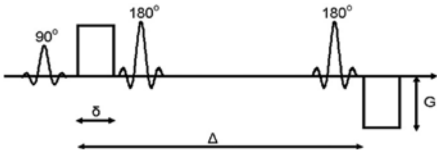


- Deviation of Gaussian distribution
- Diffusion propagator can be calculated

$$\frac{S(\gamma G\delta)}{S_0} = \int_{-\inf}^{\inf} P(R,\Delta) e^{i\gamma\delta R} dR \qquad \Delta >> \delta$$

$$\Rightarrow P(R,\Delta) = \int_{-\inf}^{\inf} S(q)\, e^{-iqR} dq \qquad q = \frac{\gamma}{2\pi} G\delta$$

Sequence for q-space imaging

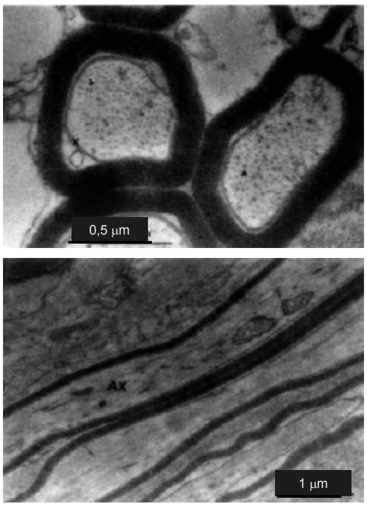


Callaghan et al. Nature 1991

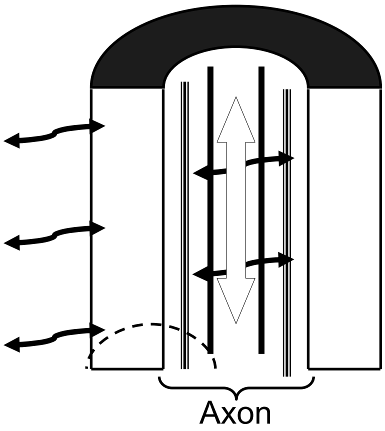
DTI

Diffusion Tensor Imaging (DTI)

Anisotropic Diffusion



Bealiue, NMR Biomed., 2002



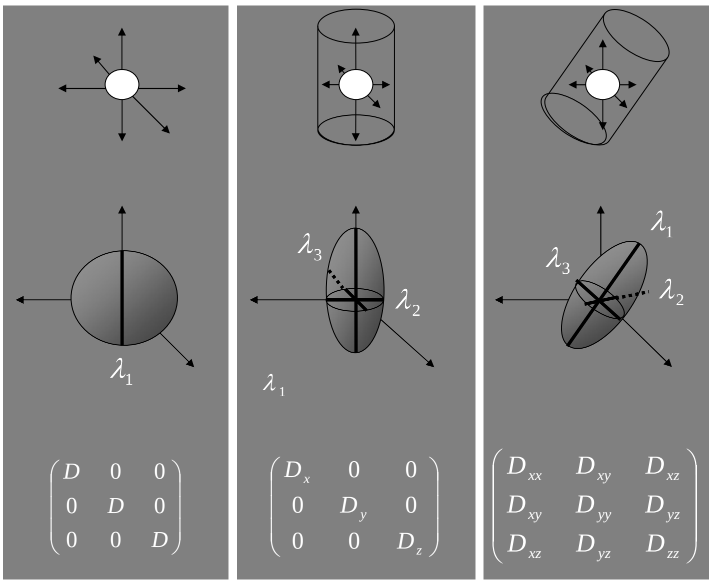
⇨ free diffusion along the fiber
restricted orthogonal to the fiber

Diffusion Tensor Imaging (DTI)

water molecule

diffusion ellipsoid

diffusion tensor



$$\begin{pmatrix} D & 0 & 0 \\ 0 & D & 0 \\ 0 & 0 & D \end{pmatrix} \quad \begin{pmatrix} D_x & 0 & 0 \\ 0 & D_y & 0 \\ 0 & 0 & D_z \end{pmatrix} \quad \begin{pmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{xy} & D_{yy} & D_{yz} \\ D_{xz} & D_{yz} & D_{zz} \end{pmatrix}$$

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DTI: Quantification

derived quantities

apparent diffusion coefficient

$$ADC = \frac{(\lambda_1 + \lambda_2 + \lambda_3)}{3}$$

fractional anisotropy (FA)

$$FA = \sqrt{\frac{3}{2}} \frac{\sqrt{(\lambda_1 - \bar{\lambda})^2 + (\lambda_2 - \bar{\lambda})^2 + (\lambda_3 - \bar{\lambda})^2}}{\sqrt{\lambda_1^2 + \lambda_2^2 + \lambda_3^2}}$$

$0 < FA < 1$

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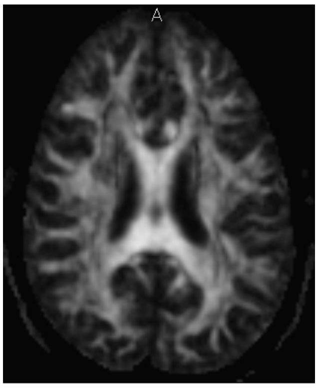
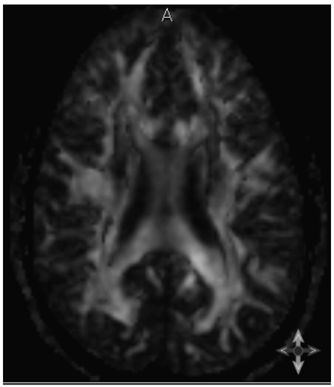
Fractional Anisotropy



FA-map



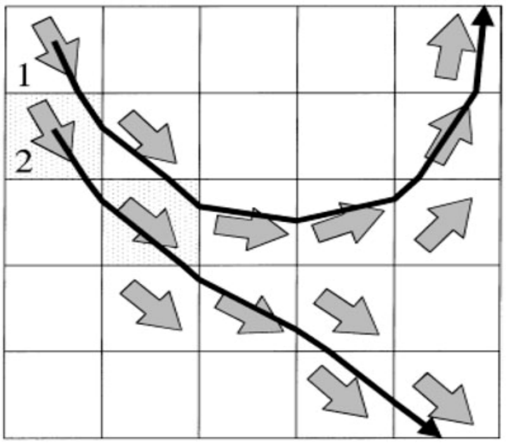
FA-colormap

white matter:
FA = 0.4 – 0.8

grey matter:
FA = 0.05 – 0.2

Fiber tracking

FACT-algorithm = fiber assignment by continuous tracking



grey arrows = eigenvector

black arrow = calculated track

stop criterion:

border of the image

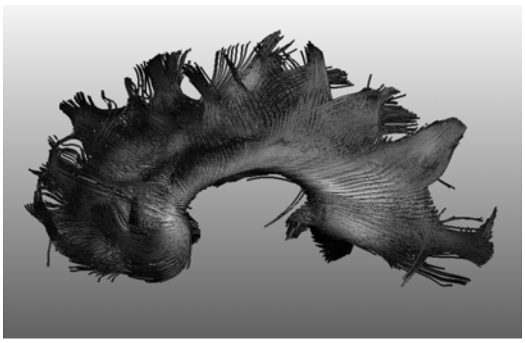
FA < 0.2 (grey matter)

high curvature

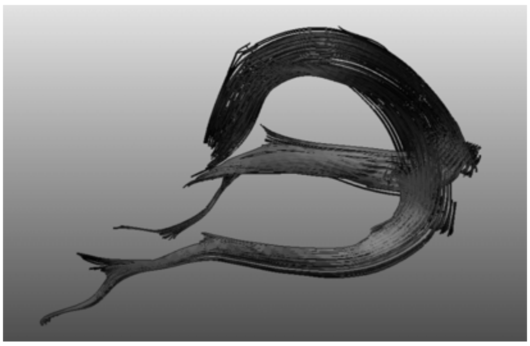
Mori et al, NMR Biomed., 2002

Fiber tracking: Examples

Corpus Callosum

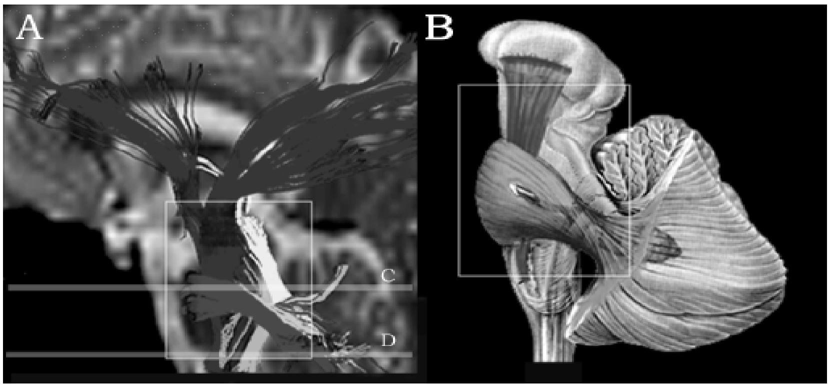


Fornix



3D Fibertracking

fibertracking method:
- using main eigenvector of diffusion tensor
- the main direction of fibers can be tracked pixel by pixel



in vivo DTI | post mortem atlas

Stieltjes et al. Neuroimage 2001