Master‘s Program in Medical Physics

Biomedical Engineering

Blood Pressure / Blood Flow

Patrick Heiler

Computer Assisted Clinical Medicine
Faculty of Medicine Mannheim
University of Heidelberg
Theodor-Kutzer-Ufer 1-3
D-68167 Mannheim, Germany
Patrick.Heiler@MedMa.Uni-Heidelberg.de

Repetition

- definition of Fourier Transform and its main properties:
  - linearity, convolution theorem, differential and integral equations in the Fourier space
- electrical measurement values
  - voltage, current, power, different impedance: Ohm, Capacitor, Inductivity (+their symbols), symbols for current or voltage sources
  - Kirchhoff rules (node and mesh rule) and being able to apply them to different electronic circuits

Overview

- Fluid Parameters: Pressure, Flow
- Fluids in Motion
- Flow of Fluids in Tubes
- Blood Pressure
- Measurement of Blood Pressure
- Pressure Sensor

Fluid Parameters

parameter	**Formula/Symbol**	**SI unit**
Density:	$\rho=m/V$	$[kg/m^3]$
Temperature	T	[K]
Velocity	$\mathbf{v} = \frac{d\,\mathbf{s}}{dt} = \begin{pmatrix} u \\ v \\ w \end{pmatrix}$	[m/s]
Pressure	p=F/A	$[N/m^2]$

Pressure Conversion Factors

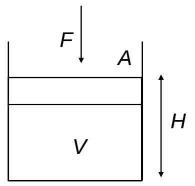
	Atmosphere	N/m^2 = Pa	mm Hg	mm H_2O
Atmosphere	1	1.01×10^5	760	10300
N/m^2 = Pa	9.87×10^{-6}	1	0.0075	0.102
mm Hg	0.00132	133	1	13.6
mm H_2O	9.68×10^{-5}	9.81	0.0735	1

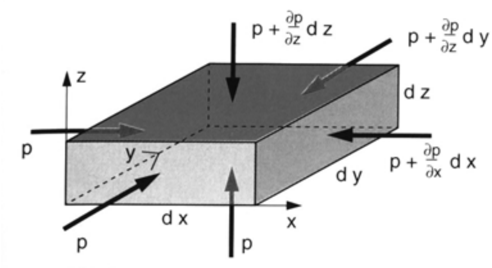
Pressure in the Body

Site		Pressure (mm Hg)
Arterial blood pressure:	systole	100–140
	diastole	60–90
Capillary blood pressure:	arterial end	~30
	venous end	~10
Venous blood pressure:	smaller veins	3–7
	great veins	<1
Cerebrospinal pressure in brain (lying down)		5–12
Gastrointestinal pressure		10–20
Bladder pressure		5–30
Lungs:	during inspiration	minus 2–3
	during expiration	2–3
Intrathoracic cavity (between lung and chest wall)		minus 10
Joints in skeleton		up to 10 000
Foot pressure:	static	up to 1200
	dynamic	up to 7500
Eye		12–23
Middle ear		<1

Static Pressure

$p_{stat} = F/A$;





$$F_x = p \cdot dydz - \left(p + \frac{\partial p}{\partial x} dx \right) dydz = -\frac{\partial p}{\partial x} dV \qquad F_y = -\frac{\partial p}{\partial y} dV \quad , \quad F_z = -\frac{\partial p}{\partial z} dV$$

$$\vec{F} = -grad\ p \cdot dV$$

Hydrostatic Pressure

Force of Gravity of volume element dV: $dF = \rho \cdot g \cdot dV, \quad \Rightarrow \quad dp = \rho \cdot g \cdot dz$

pressure on ground element da: $p_{atm}(0) = \int_0^H \rho \cdot g \cdot dz = \rho \cdot g \cdot H$

➔ hydrostatic pressure distribution: $p_{atm}(h) = \int_h^H \rho \cdot g \cdot dz = \rho \cdot g (H - h)$

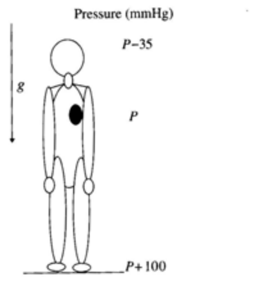
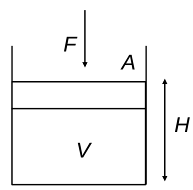


Figure 2.1. *Hydrostatic pressure when standing.*



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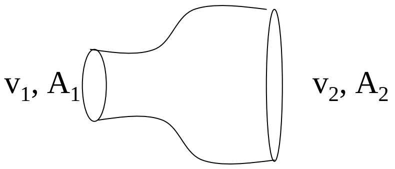
Flow

flow: Q = dV/dt = Av

⇑

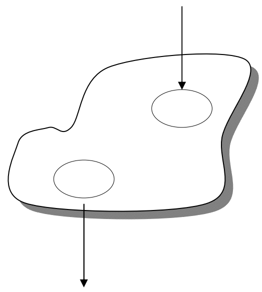
dV = A ds = A v dt



incompressible flow

continuity law: $Q = v_1 A_1 = v_2 A_2 = const$

general case: $Q = V_1 A_1 + Q_{source} – Q_{drain}$



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Bernoulli-Equation

Potential Energy: $\Delta W_1 = F_1 \Delta x_1 = p_1 A_1 \Delta x_1 = p_1 V_1$

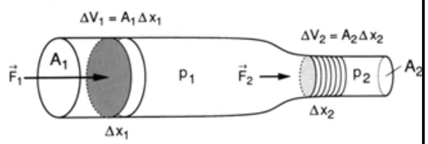
$\Delta W_2 = p_2 V_2$

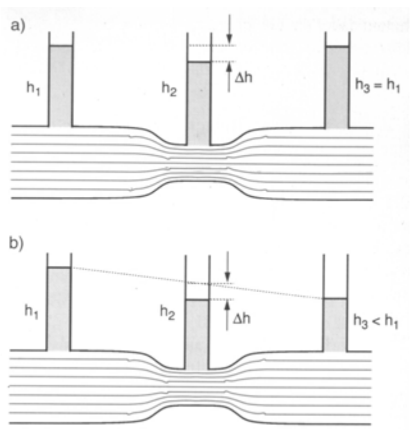
Kinetic Energy: $E_{kin} = \frac{1}{2} m v^2 = \frac{1}{2} \rho v^2 V$

Conservation of Energy:

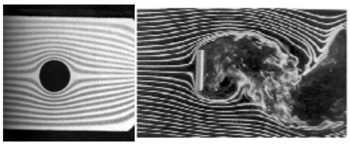
$$p_1 V_1 + \frac{1}{2} \rho v^2 V_1 = p_2 V_2 + \frac{1}{2} \rho v^2 V_2$$

$$p_1 + \frac{1}{2} \rho v^2 = p_2 + \frac{1}{2} \rho v^2 = const.$$



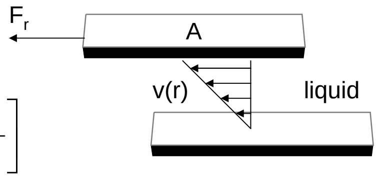


Fluids in Motion I



$$F_F = \eta \cdot A \frac{dv}{dr}$$

η ... viscosity $\left[\frac{NS}{m^2}\right]$

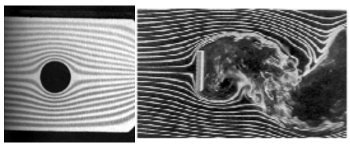


Examples:

fluid	η [mNs/m²]=[mPas]
water	1.002
blood	approx. 3-5
glycerin	1480
mercury	1.55

T = 20°

Fluids in Motion II



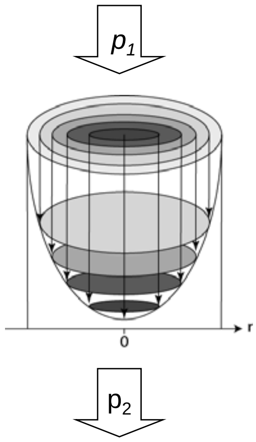
laminar current: frictional force > accelerating Force

otherwise formation of turbulences possible (where velocity gradient is strong)

distinction: Reynolds number: $\mathrm{Re} = \frac{\rho \cdot v \cdot D}{\eta}$ D ... characteristic length
... tube diameter

- laminar flow: Re < 2300
- transient: 2300 < Re < 4000
- turbulent flow: Re > 4000

Flow of Fluids in Tubes I



Due to symmetry the velocity of the current is only depended on the distance to the axis!

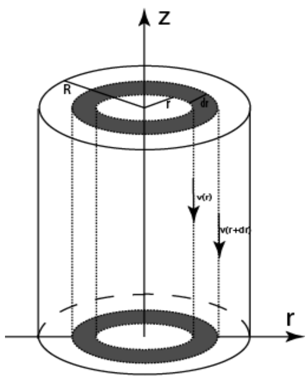
Frictional force and resulting pressure force on the end surfaces are in equilibrium.

➔ $$F_F = 2\pi r L \cdot \eta \frac{dv}{dr} = \pi r^2 (p_1 - p_2) = F_p$$

⇔ $$\frac{dv}{dr} = \frac{p_1 - p_2}{2\eta L} r$$

➔ $$v(r) = \int_r^R \frac{p_1 - p_2}{2\eta L} r dr = \frac{p_1 - p_2}{4\eta L} \cdot (R^2 - r^2)$$

Flow of Fluids in Tubes II



Flow: $$Q = \int_0^R v(r) dA = \int_0^R v(r) 2\pi r dr$$

$$= \int_0^R \frac{2\pi r (p_1 - p_2)}{4\eta L} (R^2 - r^2) dr$$

$$= \frac{\pi R^4 (p_1 - p_2)}{4\eta L} - \frac{\pi R^4 (p_1 - p_2)}{8\eta L}$$

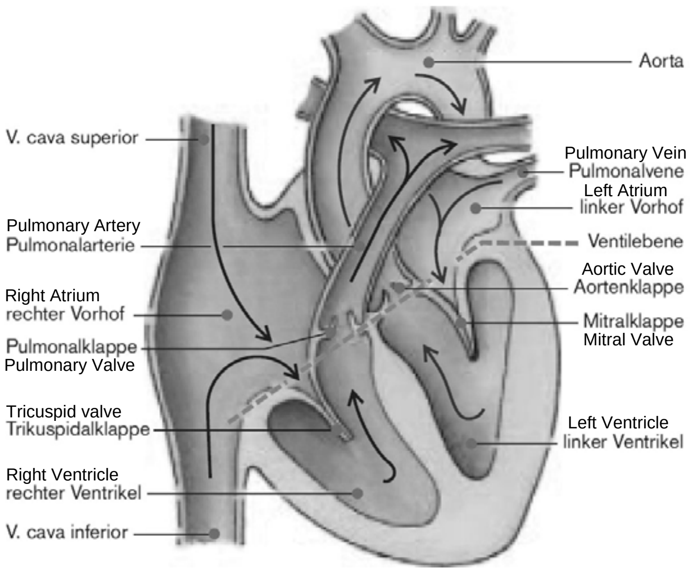
$$= \frac{\pi R^4 (p_1 - p_2)}{8\eta L}$$ Poiseuille´s equation

<u>Resistance:</u> $$K = \frac{p_2 - p_1}{Q} = \frac{8\eta L}{\pi R^4}$$

Physiological Basis

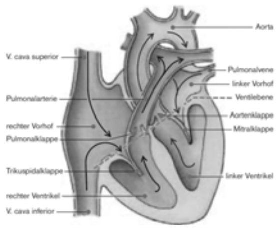
- blood: incompressible, laminar liquid
- pressure generator: heart
- cardiac output: volume that is pumped from one ventricle per time (minute)
- pumped volume per beat: approx. 70 cm^3
- 60 beats per minute= 4,2 l/min

The Heart

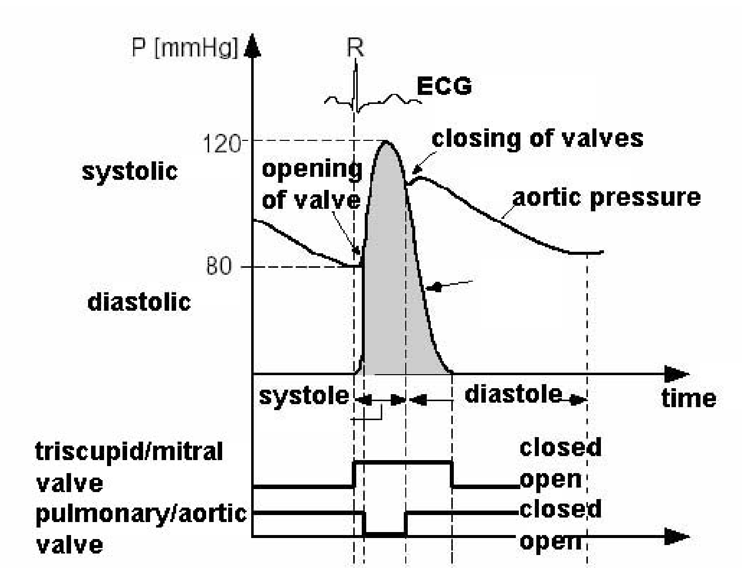


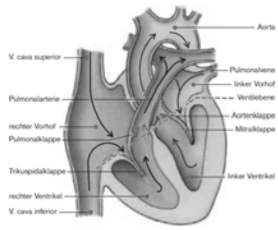
Blood Pressure in Arterial Part

- 4 phases
  - strain phase
    - chamber filled, contraction, closing by valve
    - increase of pressure: 0,27-1,47 kPa to 10,7 kPa (2-11 to 80 mm Hg)
  - ejection phase
    - valve opens
    - max. pressure: 16 kPa (120 mm Hg)
  - relaxation phase
  - ventricle pressure < aortic pressure
    - valve closes, ventricle pressure approx. 0 mm Hg, aortic pressure approx. const.
  - filling phase
    - valves open, increase in pressure
    - dynamic area: 80-120 mm Hg

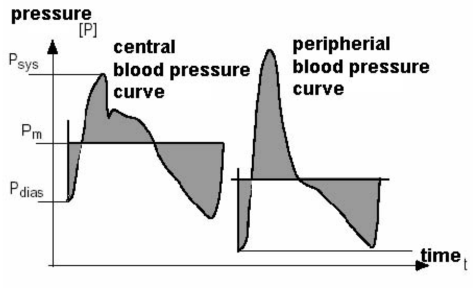


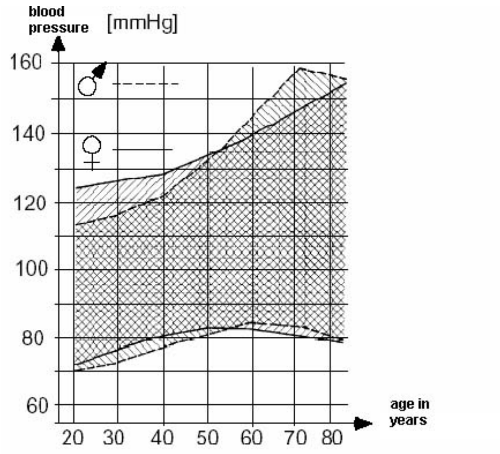
Heart Phases





Arterial Blood Pressure





Blood Pressure and Influences

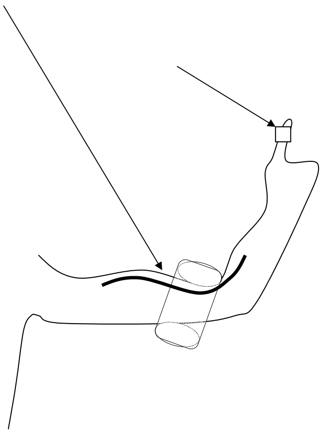
- **normal values**
  - systolic pressure: age in years + 100
  - diastolic pressure: 90
- **depends from**
  - respiration (fast breast breathing: increasing, deep abdominal: increasing)
  - physical stress
  - physical factors (sleep, filling of bladder (increases with filling), after eating (higher))
  - external temperature (colder = higher)
  - body temperature
  - time of day (minimum at 3 am, maximum at 3 pm)
  - muscle load
  - body position (lying: low, standing: high)
  - blood loss (decreasing)
  - measurement location
  - age (increasing)

Blood Pressure and Influences

- pathological: systolic pressure > 160, diastolic pressure > 95
  - higher risk for defects of coronary arteries/heart infarct, brain infarct (cerebral apoplexy), damages of nerves, aneurisms
  - hypertension: average blood pressure over 100 mm Hg

Measurement of Blood Pressure

- Riva Rocci
  - upper arm: fix cuff (arteria brachialis)
  - P > p_{sys} (200 mm Hg)
  - no blood flow: photo sensor at finger
  - pulsing flow: systolic pressure
  - continuous flow: diastolic pressure



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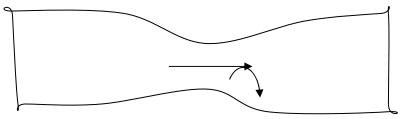
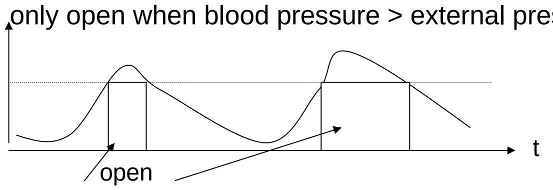
Korotkoff I

Remember: $\mathrm{Re} = \frac{\rho \cdot v \cdot D}{\eta}$

- reduced cross section
  - higher velocity v
  - Re larger: turbulence
  - curls: bump against wall: noise, pressure variation



  - only open when blood pressure > external pressure



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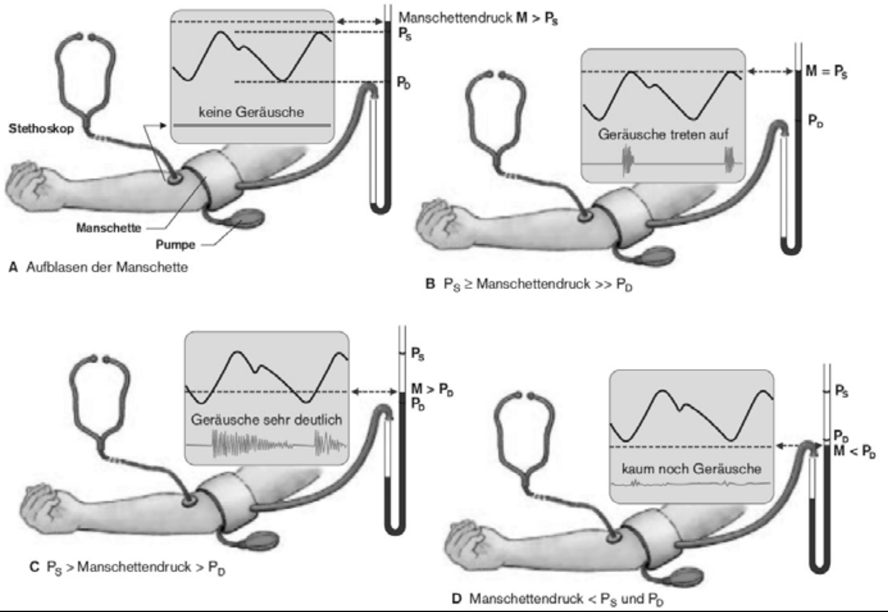
Korotkoff II



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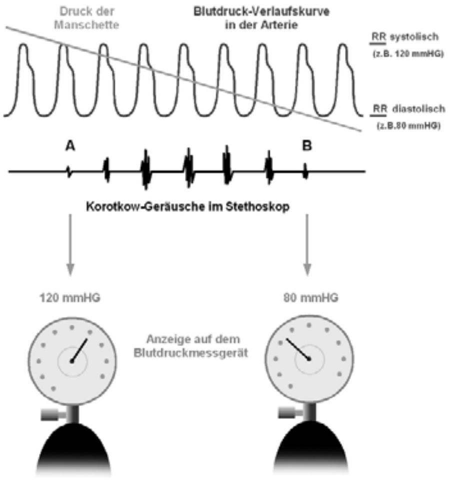
Korotkoff III



- Problems:
  - Cuff must be at heart level to obtain a pressure that is uninfluenced by gravity
  - Just possible to measure pressure in the arm (arteria brachialis)
  - If cuff is left inflated for some time, the discomfort may cause an reflex raising the blood pressure
  - Incorrect size of cuff
  - Incorrect speed of pressure reduction
  - Incorrect placement of cuff and stethoscope membrane
  - Difficulty to distinguish between continuous and staccato sound

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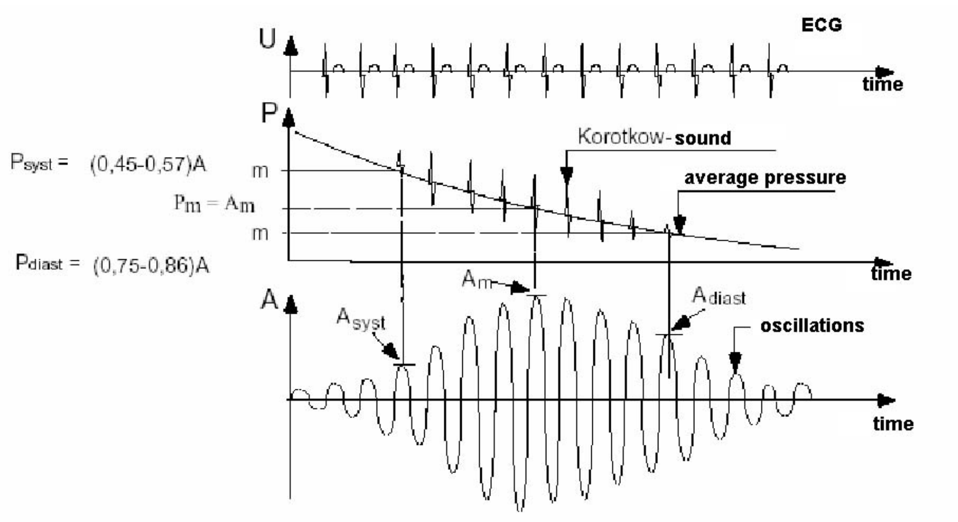
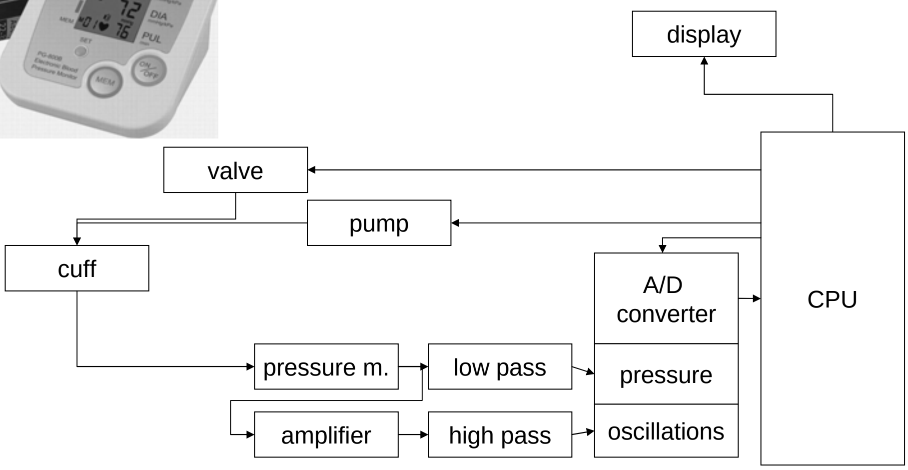
Oscillometric Blood Pressure Measure I

- same principle
- during measurement of pressure at cuff: oscillations
- empirical criteria for systolic and diastolic pressure
  - determine amplitudes of oscillations
  - average pressure: cuff pressure where maximal amplitude occurs
  - remainder over specific algorithms like
  - A: amplitude; A_{sys} = 45-57% of A_m (normal value 50%): first occurrence of Korotkoff-sound
  - A_{dia} = 75-86% (normal value 80%) of Am: Korotkoff-sound vanishes

Oscillometric Blood Pressure Measure II



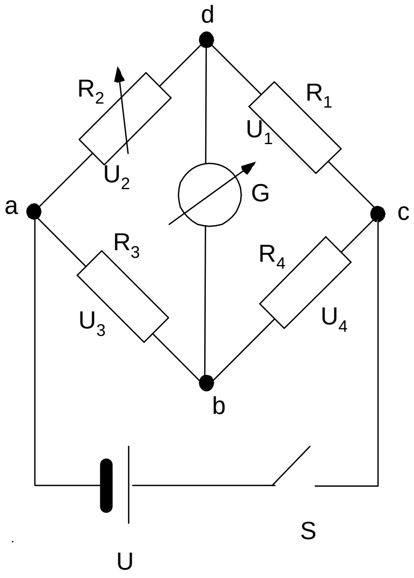
Oscillometric Blood Pressure Measure III



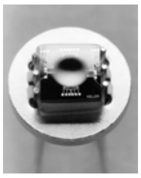
Problems

- motion artifacts (pressure reduction in steps, 2 measurements per step, if the same: no disturbance
- advantages
  - no acoustic measurements
  - good artifact detection and suppression
  - also applicable for newborns and babies

Pressure Sensor: Potentiometric Sensors



$$R = \rho \frac{l}{A} \propto \Delta p$$



$$\frac{U_1}{U_2} = \frac{R_1}{R_2} \quad \frac{U_4}{U_3} = \frac{R_4}{R_3} \quad \text{(same currents)}$$

no current through G: $U_1 = U_4; U_2 = U_3$

$$\frac{R_2}{R_1} = \frac{R_3}{R_4}$$

$$R_1 = R_2 \frac{R_4}{R_3}$$

Questions

1. Will blood pressure measured on the arm be higher or lower if the arm is raised slightly?
2. What is the most important distinction between mechanics of solids and fluids?
3. How does the resistance of blood flow depend upon the effective diameter of a blood vessel?
4. Does the Reynolds number decrease or increase with distance from the heart?
5. According to the Poisseuille equation to flow rate, Q in a tube is proportional to what powers of the radius and length?

$$Q \propto R^a L^b$$