



Biomedical Engineering

Blood Pressure / Blood Flow

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- continuation law, hydrostatic pressure, Hagen-Poiseuille, Bernoulli-equation, Reynolds number
- Riva-Rocci, Korotkow, oscillatory blood pressure measurement



Blood Pressure Measurement

Non-Invasive Blood Pressure Measurement:

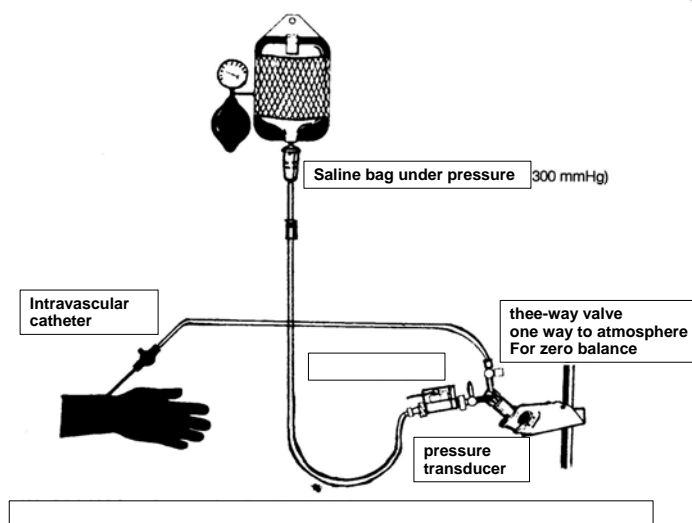
- Soft tissue: signal damping
- Only big vessels close to surface measurable
- Poor temporal resolution
- Continuous measurements not possible

Invasive Blood Pressure Measurement:

- Direct connection between blood and sensor
- Improved accuracy and temporal resolution

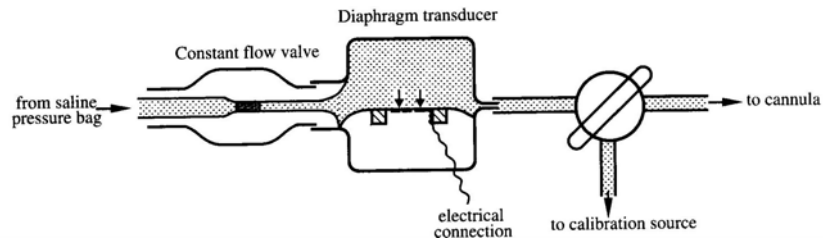


Invasive Blood Pressure Measurement I

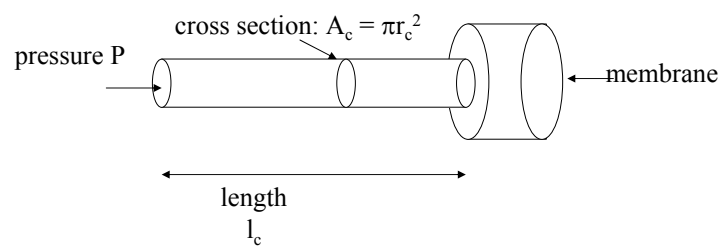




Invasive Blood Pressure Measurement II



Invasive Blood Pressure Measurement III



compliance: $C_p = \frac{\Delta V}{\Delta p}$

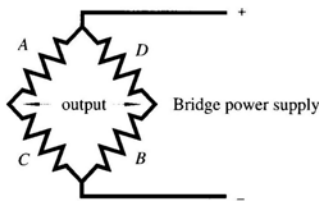
acceleration: $a_c = \frac{F}{m} = \frac{\Delta P A}{m}$

displacement: $\Delta x = \frac{\Delta V}{A} = \frac{C_p \cdot \Delta p}{A}$

time: $\Delta t = \sqrt{\frac{2\Delta x}{a_c}} = \sqrt{\frac{2C_p \cdot \Delta p \cdot m}{\Delta p \cdot A}} = \sqrt{\frac{2C_p \cdot \rho \cdot l}{A}}$



Resistance Bridge



Strain gauges A and B have a positive strain

Strain gauges C and D have a negative strain

Resistance:

$$R_{A,B} = R_0 - ks + cT$$

$$R_{C,D} = R_0 + ks + cT$$

$$\phi_{AC} = U \frac{R_c - R_A}{R_c + R_A} \quad \phi_{BD} = U \frac{R_B - R_D}{R_B + R_D}$$

$$U_{out} = \phi_{AC} - \phi_{BD} = \frac{2ks}{2R_0 + 2ct} - \frac{-2ks}{2R_0 + 2ct} = \frac{2ks}{R_0 + ct}$$



Procedure for Pressure Transducer

1. Make the electrical connection between the transducer and the measurement equipment.
2. Connect the pressure lines, with the exception of the patient connection.
3. Fill the system with saline and remove all the air bubbles.
4. Open the transducer to the atmosphere using the three way tap.
5. Zero the bridge and amplifier so that zero output corresponds to atmospheric pressure.
6. Close the tap to atmosphere and now make the patient connection
7. Flush a small volume of saline through the system and check that the blood is not entering the system.
8. The system should now be ready for measurement

Dynamic Performance

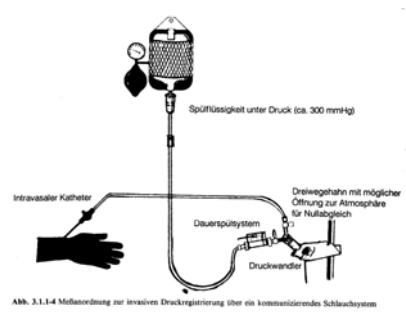
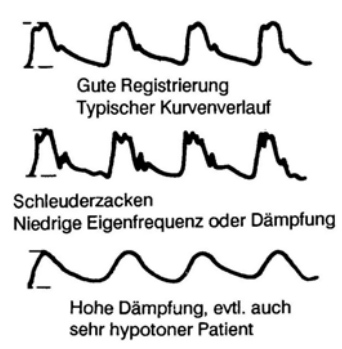
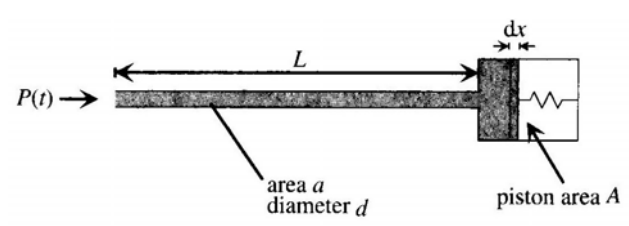


Abb. 3.1.1-4 Messanordnung zur invasiven Druckregistrierung über ein komprimierendes Schlauchsystem



Dynamic Performance



to avoid standing waves the length should be $l_c < \lambda/4$

$$F(t) = Dx + m\ddot{x} + r\dot{x}$$

- $F_H = Dx$ Hook law
- $F_N = ma$ 2nd Newton Law
- $F_F = rv$ friction



Analogy I

$$F(t) = Dx + m\ddot{x} + r\dot{x}$$

$$v(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} V(\omega) e^{i\omega t} d\omega,$$

$$V(\omega) = \int_{-\infty}^{\infty} v(t) e^{-i\omega t} dt$$

Fourier space:

$$F(\omega) = D \frac{V}{i\omega} + i\omega m V + rV$$

$$Z_{mech} = \frac{F}{V} = \frac{D}{i\omega} + i\omega m + r$$

Remember:

Ohm, capacitive, and inductive resistor
in series

$$U(t) = \frac{Q}{C} + Li + Ri$$

$$\tilde{U} = \frac{1}{i\omega C} \tilde{I} + i\omega L \cdot \tilde{I} + R\tilde{I}$$

$$Z_{el} = \frac{1}{i\omega C} + i\omega L + R$$



Analogy II

force F

voltage U

pressure P

velocity v

current I

flow Q

mass m

inductivity L

effective mass m'

compliance/softness n
(1/D)

capacity C

compliance C_v

friction r

resistance R

flow resistance r_f



Example I

$$m = \rho V \quad v = \frac{\Delta V}{A \Delta t}$$

$$E_{kin} = m \frac{v^2}{2} = \frac{1}{2} \rho A l \left(\frac{\Delta V}{A \Delta t} \right)^2 = \frac{\rho l \left(\frac{\Delta V}{\Delta t} \right)^2}{2A} = m' \frac{Q^2}{2}$$

$$V = A l \quad m' = \frac{\rho l}{A}$$



Example II

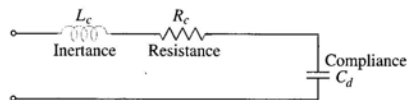
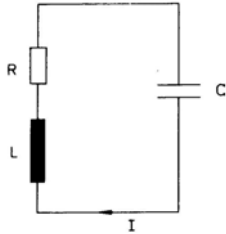


Figure 18.8. Simplified electrical model of transducer and catheter.



LC – Oscillator I



$$L\dot{I} + RI + \frac{Q}{C} = 0 \quad I(t) = \dot{Q}(t) = C\dot{U}(t)$$

$$LC\ddot{U}(t) + RC\dot{U} + U = 0$$

Damping factor: $2\beta = \frac{R}{L}$,

Natural frequency: $\omega_0^2 = \frac{1}{LC}$

$$\ddot{U} + 2\beta\dot{U} + \omega_0^2 U = 0 \quad \rightarrow \quad \omega = -\beta \pm \omega' \quad \omega' = \sqrt{\omega_0^2 - \beta^2}$$

$$-\omega^2 + 2\beta i\omega + \omega_0^2 = 0 \quad U(t) = e^{-\beta t} (Ae^{i\omega t} + Be^{-i\omega t})$$

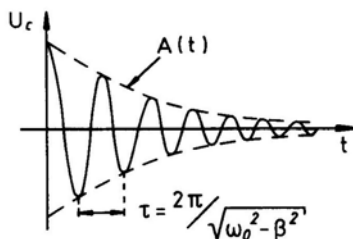
$$A = \frac{1}{2}U_0(1 - i\frac{\beta}{\omega}), \quad B = \frac{1}{2}U_0(1 + i\frac{\beta}{\omega})$$



LC – Oscillator II

$$\omega = -\beta \pm \sqrt{\omega_0^2 - \beta^2} \quad U(t) = e^{-\beta t} (Ae^{i\omega t} + Be^{-i\omega t})$$

Weak damping: $\beta^2 < \omega_0^2 \Leftrightarrow R^2 < 4\frac{L}{C}$



$$U(t) = A(t) \sin(\omega t + \varphi)$$

$$A(t) = U_0 \frac{\omega}{\omega_0} e^{-\beta t}$$

LC – Oscillator III



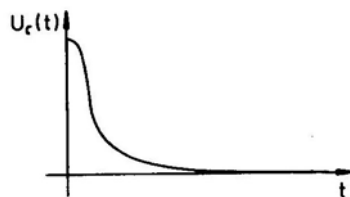
$$\omega = -\beta \pm \sqrt{\omega_0^2 - \beta^2}$$

$$U(t) = e^{-\beta t} (Ae^{i\omega t} + Be^{-i\omega t})$$

Critical damping: $\beta^2 = \omega_0^2$

$$U(t) = \alpha(t) \cdot e^{-\beta t}$$

$$\alpha(t) = U_0(1 - \beta t)$$



LC – Oscillator IV



$$\omega = -\beta \pm \sqrt{\omega_0^2 - \beta^2}$$

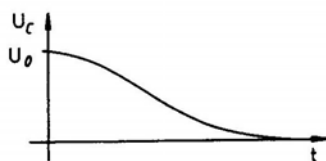
$$U(t) = e^{-\beta t} (Ae^{i\omega t} + Be^{-i\omega t})$$

Strong damping:

$$\beta^2 > \omega_0^2 \Leftrightarrow R^2 > 4 \frac{L}{C}$$

$$\gamma = i\omega$$

$$U(t) = e^{-\beta t} (Ae^{-\gamma t} + Be^{\gamma t})$$





Resonance

$$\tilde{U} = \frac{1}{i\omega C} \tilde{I} + i\omega L \cdot \tilde{I} + R\tilde{I}$$

$$Z_{el} = \frac{1}{i\omega C} + i\omega L + R = R + i\left(\omega L - \frac{1}{\omega C}\right)$$

Maximum Amplitude: $I_0 = \frac{U_0}{|Z|} = \frac{U_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$

Resonance frequency: $\omega = \omega_0 = \frac{1}{\sqrt{LC}}$



Analogy I

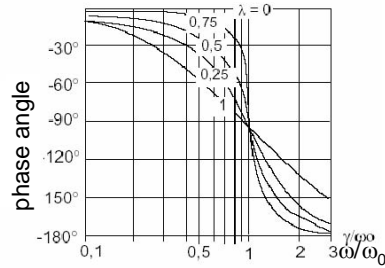
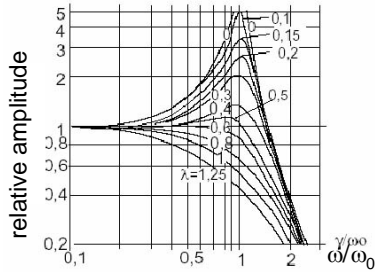
Catheter: Resistance: $r_f = \frac{\Delta p}{lQ} = \frac{8\eta}{\pi \cdot r^4}$ Poiseuille's equation $Q = \frac{\pi r^4 \Delta p}{8\eta l}$

Resonance frequency: $\omega_0^2 = \frac{1}{LC} = \frac{1}{m'C_p} = \frac{\Delta P}{m'\Delta V} = \frac{A\Delta P}{\rho l\Delta V} = \frac{\pi r^2 \Delta P}{\rho l\Delta V}$

Damping: $2\beta = \frac{R}{L} = \frac{r_f}{m'} = \frac{8\eta l}{\pi \cdot r^4} \cdot \frac{\pi r^2}{\rho l} = \frac{8\eta}{\rho r^2}$



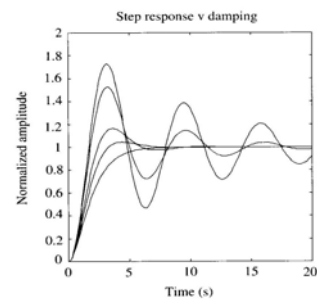
Analogy II



$$\lambda = \frac{\beta}{\omega_0}$$

choose damping so that amplitude = 1

$$f_0 \gg f_{\max}$$

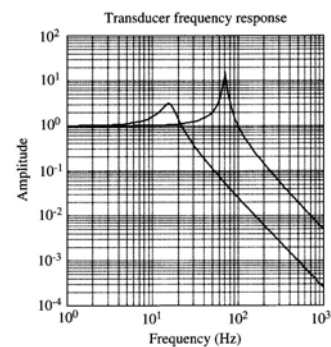


Influence of Air Bubbles

Rise in Pressure → Compression of the air bubble

Compliance of the air bubble: $C_B = \frac{\Delta V}{\Delta P} \cdot [\text{Volume of air bubble}]$

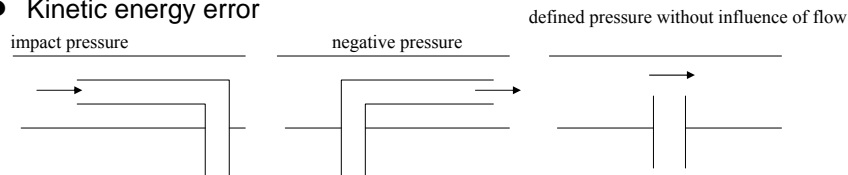
- Example:**
- 2 m long stiff catheter
 - radius 0.5 mm
 - 5 mm long air bubble





Change of Measurement Conditions

- blood agglutination: smaller radius (in tube)
- coupling fluid is contaminated by blood: viscosity and density change (good seals needed)
- air bubbles in catheter: compliance changes
- cross section of catheter changes
- change of position: measurement not perpendicular to flow direction: impact pressure
- height difference
- Kinetic energy error



Questions

- Is a high compliance good or bad in a pressure transducer?
- What effect do air bubbles have on the frequency response of a fluid filled catheter pressure measurement system?
- How will increasing the diameter of a catheter affect the resonant frequency and damping of a pressure measurement system?
- Will a catheter pressure system underestimate or overestimate the true arterial pressure if the catheter is facing downstream?
- Why there are four strain gauges on the diaphragm of a pressure transducer?

Blood Flow Amplitude and Resistance



- properties of blood
 - viscous
 - incompressible
 - laminar
 - Hagen-Poiseuille law is valid
 - radius dependence
 - change of vessel compliance: vessels become stiffer, smaller radius therefore heart generates a higher pressure

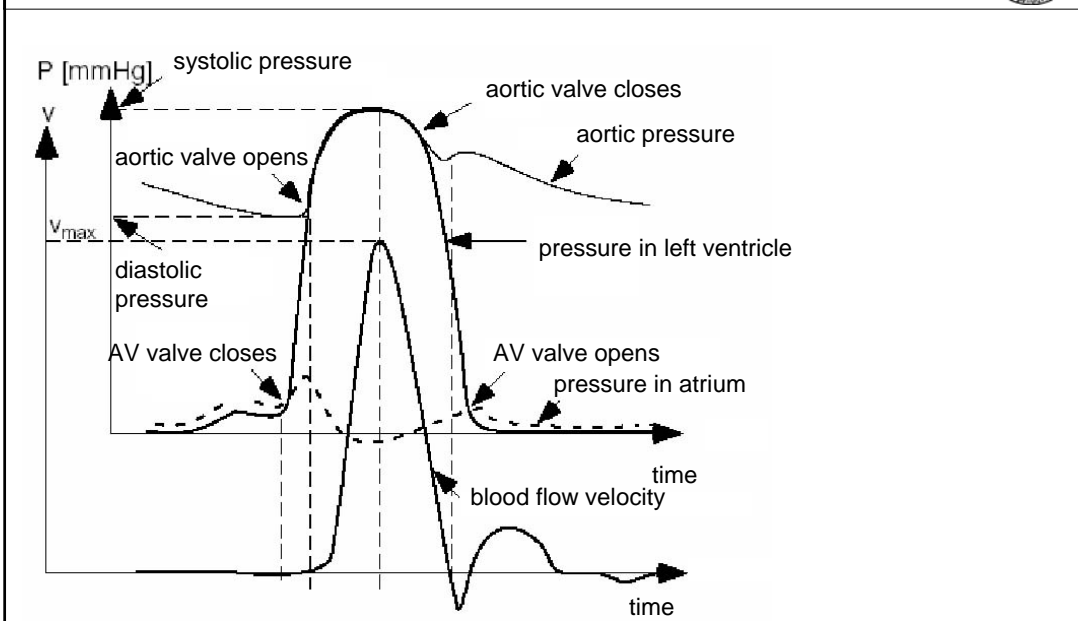
Physiological Basis



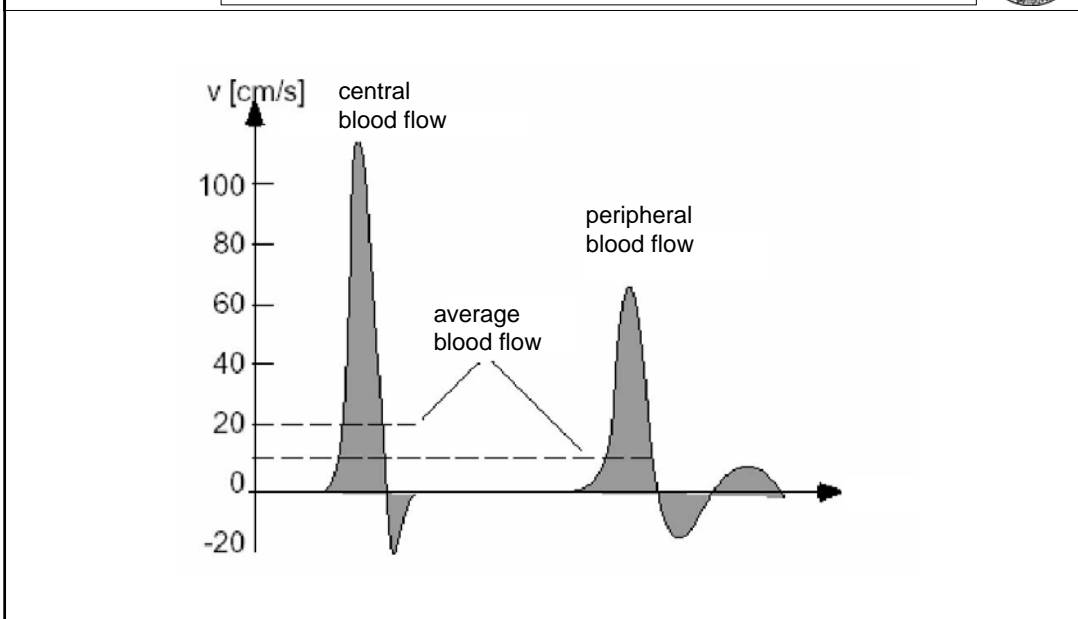
vessel	cross section [mm]	contribution to peripheral resistance [%]	average velocity [cm/s]	average blood pressure [mmHg]
aorta	20-25	0	20	100
small arteries	-	19	2-10	70-95
arterioles	-	47	0.2-0.3	35-70
capillary	-	27	-	-
-arterial end	0.01	-	0.03-0.1	30-35
-middle part	0.01	-	0.03-0.1	20-25
-venous end	0.01	-	0.03-0.1	15-20
venoles	0.5-1	4	0.5-1	10-15
veins	1-515	3	1-10	<10
vena cava	20-30	0	10-16	<5



Blood Flow as Function of Time I



Blood Flow as Function of Time II





Measurement

- Electromagnetic
- Ultrasound
 - Time
 - Doppler



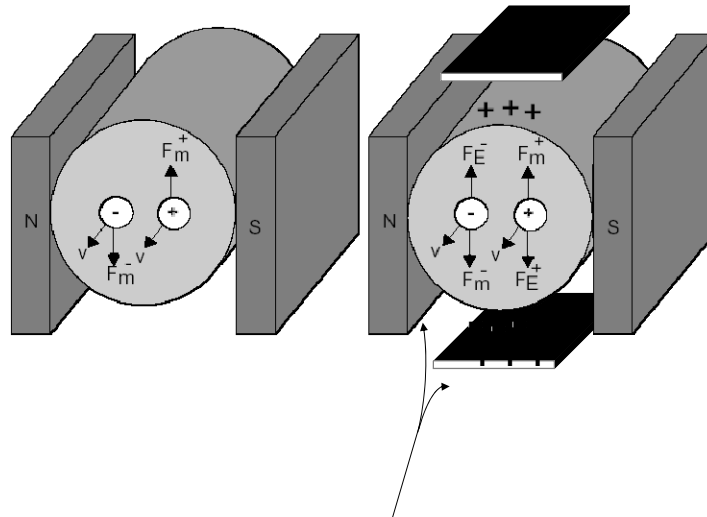
Electromagnetic Flowmeters I

- idea: use Hall effect
- basis:
 - B-field in vessel
 - Lorentz force at ions:

- direction given by sign of charge



Electromagnetic Flowmeters II



Hall Effect



- separate charge: generates electric field:

$$\vec{F}_{el} = q\vec{E}$$

- Hall effect: equilibrium between both forces

$$\vec{F}_{el} = \vec{F}_{Lorenz}$$

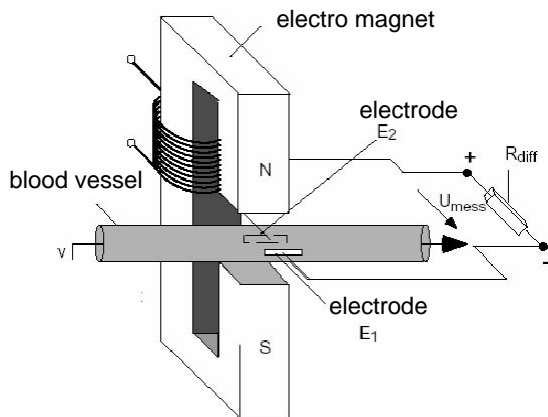
$$qE = qvB \cdot \sin(\vec{v}, \vec{B})$$

$$\vec{v} \perp \vec{B} \Rightarrow qE = q \frac{U_{Hall}}{d} = qvB$$

$$v = \frac{U_{Hall}}{Bd}$$

$$Q = Av \Rightarrow Q = \frac{\pi r^2 U_{Hall}}{Bd} = \frac{\pi r U_{Hall}}{2B}$$

Measurement Device



Example



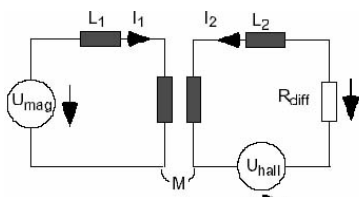
- Vessel Diameter: $r = 10 \text{ mm}$
- Magnetic Field: $B = 0.025 \text{ T}$
- Blood Velocity: $v = 1 \text{ m/s}$

- $$U_{Hall} = v \cdot B \cdot d = 1 \frac{m}{s} \cdot 0.025 T \cdot 0.01 m = 250 \mu \frac{m \cdot Vs}{s \cdot m^2} = 250 \mu V$$



Noise I

- polarization voltage at electrode for constant magnetic field: current
 - use alternating fields: 400Hz
 - smaller: transformer too large
 - larger: too large scatter capacity
 - leads to disturbing currents in wire loop



$$U_{\text{mag}} = i\omega L_1 I_1 + i\omega M I_2$$

$$U_{\text{meas}} = i\omega M I_1 + i\omega L_2 I_2 + U_{\text{Hall}} = -R_{\text{diff}} I_2$$

$$I_2 \approx 0 \text{ (voltage measurement without current)} \Rightarrow$$

$$U_{\text{mag}} = i\omega L_1 I_1$$

$$U_{\text{meas}} = i\omega M I_1 + U_{\text{Hall}} = U_{\text{mag}} \frac{M}{L_1} + U_{\text{Hall}}$$



Noise II

$$U_{\text{meas}} = U_{\text{mag}} \frac{M}{L_1} + U_{\text{Hall}}$$

- problem: U_{mag} depends on $\frac{dB}{dt}$: should be as small as possible

$$U_{\text{mag}} = \frac{d}{dt} (B \cdot A) = \left(\frac{dB}{dt} \right) A$$

- use rectangle pulses and don't measure at edges

