Overview

- Fluid Parameters: Pressure, Flow
- Fluids in Motion
- Flow of Fluids in Tubes
- Blood Pressure
- Measurement of Blood Pressure
- Pressure Sensor
### Fluid Parameters

<table>
<thead>
<tr>
<th>parameter</th>
<th>Formula/ Symbol</th>
<th>SI unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density:</td>
<td>$\rho = \frac{m}{V}$</td>
<td>$[\text{kg/m}^3]$</td>
</tr>
<tr>
<td>Temperature</td>
<td>$T$</td>
<td>$[\text{K}]$</td>
</tr>
<tr>
<td>Velocity</td>
<td>$\nu = \frac{ds}{dt} = \begin{pmatrix} \nu_x \ \nu_y \ \nu_z \end{pmatrix}$</td>
<td>$[\text{m/s}]$</td>
</tr>
<tr>
<td>Pressure</td>
<td>$p = \frac{F}{A}$</td>
<td>$[\text{N/m}^2]$</td>
</tr>
</tbody>
</table>

### Pressure Conversion Factors

<table>
<thead>
<tr>
<th></th>
<th>Atmosphere</th>
<th>$\text{N/m}^2 = \text{Pa}$</th>
<th>$\text{mm Hg}$</th>
<th>$\text{mm H}_2\text{O}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atmosphere</td>
<td>1</td>
<td>$1.01 \times 10^5$</td>
<td>760</td>
<td>10300</td>
</tr>
<tr>
<td>$\text{N/m}^2 = \text{Pa}$</td>
<td>$9.87 \times 10^{-6}$</td>
<td>1</td>
<td>0.0075</td>
<td>0.102</td>
</tr>
<tr>
<td>$\text{mm Hg}$</td>
<td>$0.00132$</td>
<td>133</td>
<td>1</td>
<td>13.6</td>
</tr>
<tr>
<td>$\text{mm H}_2\text{O}$</td>
<td>$9.86 \times 10^{-5}$</td>
<td>9.81</td>
<td>0.0735</td>
<td>1</td>
</tr>
</tbody>
</table>
Pressure in the Body

<table>
<thead>
<tr>
<th>Site</th>
<th>Pressure (mm Hg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arterial blood pressure:</td>
<td></td>
</tr>
<tr>
<td>systolic</td>
<td>100-140</td>
</tr>
<tr>
<td>diastolic</td>
<td>60-90</td>
</tr>
<tr>
<td>Capillary blood pressure:</td>
<td></td>
</tr>
<tr>
<td>arterial end</td>
<td>~30</td>
</tr>
<tr>
<td>venous end</td>
<td>~10</td>
</tr>
<tr>
<td>Venous blood pressure:</td>
<td></td>
</tr>
<tr>
<td>smaller veins</td>
<td>3-7</td>
</tr>
<tr>
<td>great veins</td>
<td>&lt;1</td>
</tr>
<tr>
<td>Cerebrospinal pressure in brain (lying down)</td>
<td>5-12</td>
</tr>
<tr>
<td>Gastrointestinal pressure</td>
<td>10-20</td>
</tr>
<tr>
<td>Bladder pressure</td>
<td>5-30</td>
</tr>
<tr>
<td>Lungs:</td>
<td></td>
</tr>
<tr>
<td>during inspiration</td>
<td>minus 2-3</td>
</tr>
<tr>
<td>during expiration</td>
<td>2-3</td>
</tr>
<tr>
<td>Intraabdominal cavity (between lung and chest wall)</td>
<td>minus 10</td>
</tr>
<tr>
<td>Joints in skeleton</td>
<td>up to 10000</td>
</tr>
<tr>
<td>Foot pressure:</td>
<td></td>
</tr>
<tr>
<td>static</td>
<td>up to 1200</td>
</tr>
<tr>
<td>dynamic</td>
<td>up to 7500</td>
</tr>
<tr>
<td>Eye</td>
<td>12-23</td>
</tr>
<tr>
<td>Middle ear</td>
<td>&lt;1</td>
</tr>
</tbody>
</table>

Hydrostatic Pressure

Force of Gravity of volume element dV: \( dF = \rho \cdot g \cdot dV \), \( \Rightarrow \) \( dp = \rho \cdot g \cdot dz \)

pressure on ground element da:

\[ p_{\text{amb}}(0) = \int_{0}^{H} \rho \cdot g \cdot dz = \rho \cdot g \cdot H \]

\( \Rightarrow \) hydrostatic pressure distribution:

\[ p_{\text{amb}}(h) = \int_{h}^{H} \rho \cdot g \cdot dz = \rho \cdot g (H - h) \]
Flow

flow: \( Q = \frac{dV}{dt} = Av \)

\( dV = A \, ds = A \, v \, dt \)

**incompressible flow**

continuity law: \( Q = v_1 \, A_1 = v_2 \, A_2 = \text{const} \)

general case: \( Q = V_1 \, A_1 + Q_{\text{source}} - Q_{\text{drain}} \)

Bernoulli’s Equation

- used to determine liquid velocities by means of pressure measurements
- „ideal liquid“, e.g. no force of viscosity

Nicols & O’Rourke, McDonald’s Blood Flow in Arteries
Bernoulli’s Equation

- Equation of continuity

\[ Q = A_1 V_1 = A_2 V_2 = A_3 V_3 \]

Nicols & O’Rourke, McDonald’s Blood Flow in Arteries

Bernoulli’s Equation

- Work done on the fluid entering the tube per unit time

\[ W_P = P_i A_i V_i \] for position C

- Liquid in motion has also kinetic energy

\[ m = \rho A_i V_i \]
\[ W_k = \frac{1}{2} V_i^2 m = \frac{1}{2} V_i^2 \rho A_i V_i \]

Nicols & O’Rourke, McDonald’s Blood Flow in Arteries
Bernoulli’s Equation

- total work

\[ W_t = P_1 A_1 V_1 + \frac{1}{2} V_1^2 \rho A_1 V_1 \]
\[ = P_2 A_2 V_2 + \frac{1}{2} V_2^2 \rho A_2 V_2 \]
\[ = (P_1 - P_2) + \frac{1}{2} \rho \left( V_1^2 - V_2^2 \right) = C \]

Nicols & O’Rourke, McDonald’s Blood Flow in Arteries

Fluids in Motion I

\[ F_r = \eta \cdot A \frac{dV}{dr} \]

\( \eta \) ... viscosity [\text{Ns/m}^2]=[\text{mPas}]

Examples:

<table>
<thead>
<tr>
<th>fluid</th>
<th>( \eta ) [mNs/m^2]=[mPas]</th>
</tr>
</thead>
<tbody>
<tr>
<td>water</td>
<td>1.002</td>
</tr>
<tr>
<td>blood</td>
<td>approx. 3-5</td>
</tr>
<tr>
<td>glycerin</td>
<td>1480</td>
</tr>
<tr>
<td>mercury</td>
<td>1.55</td>
</tr>
</tbody>
</table>

T = 20°
Fluids in Motion II

- laminar current:
  - frictional force > accelerating force
- otherwise formation of turbulences possible (where velocity gradient is strong)

**distinction: Reynolds number:**

\[
Re = \frac{D \cdot v \cdot D}{\eta}
\]

- laminar flow: \( Re < 2300 \)
- transient: \( 2300 < Re < 4000 \)
- turbulent flow: \( Re > 4000 \)

Nicols & O'Rourke, McDonald's Blood Flow in Arteries
Flow of Fluids in Tubes I

- due to symmetry the velocity of the current is only depended on the distance to the axis!
- frictional force and resulting pressure force on the end surfaces are in equilibrium.

\[ F_f = 2\pi r L \cdot \eta \frac{dv}{dr} = \pi r^2 (p_1 - p_2) = F_r \]

\[ \frac{dv}{dr} = \frac{p_1 - p_2}{2\eta L} \cdot r \]

\[ v(r) = \int_0^R \frac{p_1 - p_2}{2\eta L} r dr + C = \frac{p_1 - p_2}{4\eta L} \cdot (R^3 - r^3) \]

Flow of Fluids in Tubes II

Flow:

\[ Q = \int_0^R v(r) dA = \int_0^R v(r) 2\pi r dr \]

\[ = \int_0^R 2\pi r (p_1 - p_2) \frac{(R^2 - r^2)}{4\eta L} dr \]

\[ = \pi R^4 (p_1 - p_2) \frac{4\eta L}{8\eta L} \]

Resistance:

\[ K = \frac{p_2 - p_1}{LQ} = \frac{8\eta}{\pi R^4} \]

Poiseuille’s equation
Physiological Basis

- blood: incompressible, laminar liquid
- pressure generator: heart
- cardiac output: volume that is pumped from one ventricle per time (minute)
- pumped volume per beat: approx. 70 cm³
- 60 beats per minute = 4.2 l/min
Blood Pressure in Arterial Part

**strain phase**
- chamber filled, contraction, closing by valve
- increase of pressure: 0.27-1.47 kPa to 10.7 kPa (2-11 to 80 mm Hg)

**ejection phase**
- valve opens
- max. pressure: 16 kPa (120 mm Hg)

**relaxation phase**
- ventricle pressure < aortic pressure
- valve closes, ventricle pressure approx. 0 mm Hg, aortic pressure approx. const.

**filling phase**
- valves open, increase in pressure
- dynamic area: 80-120 mm Hg
Arterial Blood Pressure

Blood Pressure and Influences I

- **normal values**
  - systolic pressure: age in years + 100
  - diastolic pressure: 90

- **depends from**
  - respiration (inspiration: decreasing, expiration: increasing)
  - physical stress
  - physical factors (sleep, filling of bladder (increases with filling), after eating (higher))
  - external temperature (colder = higher)
  - body temperature
  - time of day (minimum at 3 am, maximum at 3 pm)
  - muscle load
  - body position (lying: low, standing: high)
  - blood loss (decreasing)
  - measurement location
  - age (increasing)
Blood Pressure and Influences II

- pathological: systolic pressure > 160, diastolic pressure > 95
- higher risk for defects of coronary arteries/heart infarct, brain infarct (cerebral apoplexy), damages of nerves, aneurisms
- hypertension: average blood pressure over 100 mm Hg

Measurement of Blood Pressure

- Riva Rocci
  - upper arm: fix cuff (arteria brachialis)
  - \( P > p_{\text{sys}} \) (200 mm Hg)
  - no blood flow: photo sensor at finger
  - pulsing flow: systolic pressure
  - continuous flow: diastolic pressure
Korotkoff I

- reduced cross section
  - higher velocity \( v \)
  - \( \text{Re} \) larger: turbulence
  - curls: bump against wall: noise, pressure variation
  - only open when blood pressure > external pressure
Korotkoff III

Problems:
- Cuff must be at heart level to obtain a pressure that is not influenced by gravity
- Just possible to measure pressure in the arm (arteria brachialis)
- If cuff is left inflated for some time, the discomfort may cause an reflex raising the blood pressure
- Incorrect size of cuff
- Incorrect speed of pressure reduction
- Incorrect placement of cuff and stethoscope membrane
- Difficulty to distinguish between continuous and staccato sound

Oscillometric Blood Pressure Measure I

- Same principle
- During measurement of pressure at cuff: oscillations
- Empirical criteria for systolic and diastolic pressure
- Determine amplitudes of oscillations
- Average pressure: cuff pressure where maximal amplitude occurs
- Remainder over specific algorithms like
  - $A$: amplitude; $A_{sys} = 45$–57% of $Am$ (normal value 50%): first occurrence of Korotkoff-sound
  - $A_{dia} = 75$–86% (normal value 80%) of $Am$: Korotkoff-sound vanishes
Oscillometric Blood Pressure Measure II

\[ P_{\text{min}} = (0.46-0.57) \text{A} \]
\[ \bar{P} = A_m \]
\[ P_{\text{max}} = (0.75-0.86) \text{A} \]

\[ \text{average pressure} \]

\[ \text{oscillations} \]

Oscillometric Blood Pressure Measure III

- valve
- pump
- cuff
- pressure m.
- low pass
- high pass
- amplifier
- A/D converter
- pressure
- oscillations
- display
- CPU
Problems

- motion artifacts (pressure reduction in steps, 2 measurements per step, if the same: no disturbance

- advantages
  - no acoustic measurements
  - good artifact detection and suppression
  - also applicable for newborns and babies