Learning objectives

- Understanding the process of image formation
- Point spread function and its properties
- Noise
- Fourier Transformation and its properties
- Sampling Theorem
Diagnostic Imaging: Milestones

1901

W. C. Röntgen (Germany, 1845 - 1923)
discovery of X-rays

1952

Felix Bloch (USA, 1905 - 1983)
Edward M. Purcell (USA, 1912 - 1997)
development of a new precision method of nuclear magnetism (NMR)

1979

Allan M. Cormack (USA, 1924 - 1998)
Godfrey N. Hounsfield (UK, 1919 - )
development of Computer-Tomography (CT)

1991

Richard R. Ernst (CH, 1933 - )
development of high resolution magnetic resonance spectroscopy (MRS)

2003

Paul C. Lauterbur (USA, 1929 - 2007)
Peter Mansfield (UK, 1933 - )
development of magnetic resonance imaging (MRI)

Diagnostic Imaging: Pioneers

ROE  PET

“The Making of a Science”

CT  MRI

source: ECR Newsletter 1/2003
Why do we need imaging systems?
„Addiction“ to image information?

source: Siemens "100 Jahre Röntgen", 1995

Image Formation

- process to generate (form) an image

- everyday images mostly 2D
- however: objects in real world are 3D
  - eye’s retina collects 2D images
  - brain reconstructions 3D objects
Image Formation

- Can we describe this process by an equation?

\[ g = T(f) \]

- Imaging system

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Imaging Equation
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- most (medical) imaging is nonlinear
- however: assume most parts of our imaging process is linear
  - e.g. object brightness is scaled, also the brightness of the image is linear scaled
  - more important: allows to decompose complex objects into smaller ones
Point Objects

- mathematically represented by delta function $\delta$
  - zero everywhere
  - at point $p(x_0, y_0, z_0)$ becomes infinite
  - constraint so that integral is 1

$$\iiint \delta(x - x_0, y - y_0, z - z_0) \, dx \, dy \, dz = 1$$

- can be represented as well behaved function, e.g. Gaussian

Sifting Integral

- property of delta function
  - picks (sifts) out values of a function $f$

$$f(x, y, z) = \iiint f(\xi, \eta, \zeta) \delta(x - \xi, y - \eta, z - \zeta) \, d\xi \, d\eta \, d\zeta$$

$$g(x, y, z) = \iiint f(\xi, \eta, \zeta) h(x - \xi, y - \eta, z - \zeta; \xi, \eta, \zeta) \, d\xi \, d\eta \, d\zeta$$

Imaging equation in current context
Point spread function (PSF)

- function $h$ is the image of a point object
- in general: not a delta function
- imaging system produces degraded representation of the object
- function $h$ spreads more or less around point $(\xi, \eta, \zeta)$
  - therefore called point spread function

Point Spread Function (PSF)

- is always non zero
- unimodal and decreasing
- shape depends on the imaging system
- often modelled by a Gaussian

\[
h(x - \xi, y - \eta, z - \zeta) = S \frac{1}{\sqrt{2\pi n d(\xi, \eta, \zeta)^3}} e^{-\frac{(x - \xi)^2 + (y - \eta)^2 + (z - \zeta)^2}{2d(\xi, \eta, \zeta)^3}}\]
Properties of the PSF

- point sensitivity function
  - Integral over \( h \rightarrow \) total signal collected

- spatial linearity
  - PSF can be shifted \( \rightarrow \) imaging system no longer a spatial linear transformation

\[
g(x, y, z) = \iiint f(\xi, \eta, \zeta) h(x - \xi, y - \eta, z - \zeta) \, d\xi \, d\eta \, d\zeta
\]

also known as convolution equation

\[
g = f \otimes h
\]

- only one \( h \) needed to characterise the imaging system
  - two parameters can be extracted: sensitivity and full-width-half-maximum (FWHM)
Properties of the PSF

If we know the PSF and the position of the imaging system is independant, the convolution integral characterises our system.

Resolution

- given by the FWHM
- for symmetric Gaussian: FWHM = 2.35 d
- if two Gaussian are combined but spaced 1 FWHM apart, the signal dips by 6% of the maximum (see b)
- of course, other ways to measure resolution
Resolution

- line pairs per unit length
- mainly used in classical photographic imaging

Resolution

- Differences in reproducing an object photographic or digital

„Siemensstern“
digital image
photo film
Noise

- variations in images are called noise
- unpredictable, random, depends on the imaging system
- becomes an irreversible addition to the image

Examples:
- PET: Poisson noise associated to radioactive decay
- MRI: eletronic noise
- CT: detector noise + reconstruction
- US: scattering
Noise

- physiological noise of object relevant?
- small animal imaging, e.g. by MRI
- electronic noise higher than physiological noise
- solution to reduce noise?
- cooling of electronics

Noise

- for a point, noise function $n(x,y)$ with expected mean $\mu(x,y)$
  $$\mu(x,y) = \langle n(x,y) \rangle$$

- and variance
  $$\sigma_n^2(x,y) = \langle (n(x,y) - \mu(x,y))^2 \rangle$$

- correlation between adjacent points and noise

- "white noise": $\mu_n = 0$ with $\sigma_n$ independent of position,

- Added noise, so imaging equation:
  $$g(x, y, z) = \iiint f(\xi, \eta, \zeta) h(x - \xi, y - \eta, z - \zeta; \xi, \eta, \zeta) d\xi d\eta d\zeta + n(x, y, z)$$
Image Formation - Summary

- Image formation can be described mathematically
  - Assume that it is a linear process
  - Can be decomposed into point objects (delta functions)

- Point spread function describes imaging system
  - Usually a smearing of the object in image space
  - Determines resolution of the imaging system (fwhm)

- Noise
  - Additional signal to the image
  - Random, unpredictable