Physics of Imaging Systems

Basic Principles of Magnetic Resonance Imaging II

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Literature I

Dance et al.: “Diagnostic Radiology Physics”
Publisher: International Atomic Energy Agency

http://www-pub.iaea.org/books/IAEABooks/8841/
Diagnostic-Radiology-Physics-A-Handbook-for-
Teachers-and-Students

Free download !!!
Literature II

Reiser and Semmler: "Magnetresonanztomographie" Chapter 2, 2002

Literature III

Physics: Nuclei

Proton

- nuclear magnetic moment
- mechanic moment (spin)
- rotation
- charge
Spin Quantum Mechanics I

The norm of nuclear spin $I$ with $\hbar = 1.05 \cdot 10^{-34}$ Js Planck’s constant

with $m$ the magnetic quantum number and discrete energy levels $-I, -I+1, \ldots, I-1, I$ in total $2I+1$ possibilities

$E_m = \frac{\hbar^2 m^2}{2I}$

Zeeman effect

Spin Quantum Mechanics II

Magnetic moment $\mu$ is defined by nuclear spin $I$:

$\gamma$ gyromagnetic ratio
proton: $\gamma/2\pi = 42.6$ MHz/T

only nuclei with $I \neq 0$ are visible by MRI!!

analogy of nuclear magnetism
NMR Nuclei

<table>
<thead>
<tr>
<th>nucleus</th>
<th>spin /</th>
<th>gyromagnetic ratio $\gamma$ [10^6 rad s^{-1} T^{-1}]</th>
<th>natural abundance of isotope in %</th>
<th>sensitivity for $B_0 =$ const. in % (rel. to $^1$H)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^1$H</td>
<td>1/2</td>
<td>2.675</td>
<td>99.98</td>
<td>100.00</td>
</tr>
<tr>
<td>$^{19}$F</td>
<td>1/2</td>
<td>2.518</td>
<td>100.00</td>
<td>83.40</td>
</tr>
<tr>
<td>$^{23}$Na</td>
<td>3/2</td>
<td>0.708</td>
<td>100.00</td>
<td>9.27</td>
</tr>
<tr>
<td>$^{31}$P</td>
<td>1/2</td>
<td>1.084</td>
<td>100.00</td>
<td>6.65</td>
</tr>
<tr>
<td>$^2$H</td>
<td>1</td>
<td>0.410</td>
<td>0.01</td>
<td>9.60 × 10^{-1}</td>
</tr>
<tr>
<td>$^{12}$C</td>
<td>0</td>
<td>-</td>
<td>98.89</td>
<td>-</td>
</tr>
<tr>
<td>$^{13}$C</td>
<td>1/2</td>
<td>0.673</td>
<td>1.11</td>
<td>1.75 × 10^{-2}</td>
</tr>
<tr>
<td>$^{14}$N</td>
<td>1</td>
<td>0.193</td>
<td>99.63</td>
<td>1.00 × 10^{-1}</td>
</tr>
<tr>
<td>$^{16}$O</td>
<td>0</td>
<td>-</td>
<td>99.76</td>
<td>-</td>
</tr>
<tr>
<td>$^{17}$O</td>
<td>5/2</td>
<td>-0.363</td>
<td>0.04</td>
<td>1.11 × 10^{-3}</td>
</tr>
<tr>
<td>$^{35}$Cl</td>
<td>3/2</td>
<td>0.262</td>
<td>75.77</td>
<td>3.58 × 10^{-1}</td>
</tr>
<tr>
<td>$^{39}$K</td>
<td>3/2</td>
<td>0.125</td>
<td>93.26</td>
<td>4.76 × 10^{-2}</td>
</tr>
<tr>
<td>$^{25}$Mg</td>
<td>5/2</td>
<td>-0.164</td>
<td>10.00</td>
<td>2.68 × 10^{-2}</td>
</tr>
<tr>
<td>$^{43}$Ca</td>
<td>7/2</td>
<td>-0.180</td>
<td>0.14</td>
<td>6.86 × 10^{-3}</td>
</tr>
<tr>
<td>$^{33}$S</td>
<td>3/2</td>
<td>0.205</td>
<td>0.75</td>
<td>1.70 × 10^{-3}</td>
</tr>
</tbody>
</table>

MRI: 110 mol
(MRS: < 10^{-3} mmol)
MRI: 50 mmol
MRS: 40 mmol

Nuclei in an External Magnetic Field

Zeeman energy levels of nuclei with $I = 3/2$

- potential energy in external $B_0$:

$$E = -\mu_z B_0$$

$$\mu_z = \gamma \hbar m$$

$$E_m = -\gamma \hbar m B_0$$

- external RF can induce transition between energy levels if $\Delta m = \pm 1$:

$$E_{RF} = \hbar \omega_{RF}$$

$$\Delta E = \hbar \omega_0 = \gamma \hbar B_0$$

$\omega_0$: Larmor frequency

$= 64$ MHz for protons at 1.5 T

$\gamma$: gyromagnetic ratio

$= 42.6$ MHz/T for protons
**Isidor Rabi**
- Germany - Columbia 1938
- Rebuilt a molecular beam apparatus (Otto Stern)
- Detected nuclear resonance in a stream of Lithium Chloride molecules

**Edward Purcell, Torrey and Pound**
- Harvard 1946
- Applied radar technology in investigating magnetic resonance
- Achieved the first resonance in a practical sample, a block of paraffin

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**Semi Classic Description**

- Mechanic precession = reaction of spinner to external force \( G \)
- Atomic precession = intrinsic properties of proton resulting from QM

Problem solved by macroscopic quantity: \( M_0 \) magnetization
Movie: Summary

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Magnetization $M_0$ for Spin $1/2$

$B = 0$

$B = B_0$

$\sum \text{ spins} = M_0$
Boltzmann Statistic $M_0$

Boltzmann statistic protons:

$$\frac{N_{+1/2}}{N_{-1/2}} = \exp\left(\frac{\Delta E}{kT}\right) = \exp\left(\gamma h B_0 / kT\right)$$

- $k$: Boltzmann constant = $1.4 \times 10^{-23}$ J/K
- $N_{+1/2}$: number of spins parallel to $B_0$
- $N_{-1/2}$: number of spins anti-parallel to $B_0$

since $\gamma h B_0 \ll kT \rightarrow$ Taylor series:

$$\frac{N_{+1/2}}{N_{-1/2}} = 1 + \frac{\gamma h B_0}{kT} = 1.0000066 = 6.6 \text{ ppm}$$

at $B_0 = 1.0$ T and $T = 37^\circ \text{C} = 310$ K.

$$M_0 = (N_{+1/2} - N_{-1/2}) \langle \mu_z \rangle / \text{volume} = (N/V)\left(\gamma^2 h^2 / 4kT\right) B_0$$

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Energy Level Ratio

(source: Lissner and Seidener. "Klinische Kernspintomographie" 1987)
Zeeman Effect

Curie’s law: \[ M_0 = \frac{\rho \cdot I(I+1) \gamma^2 \hbar^2 B_0}{3kT} \]

- \( m = -\frac{1}{2} \)
- \( m = +\frac{1}{2} \)

splitting of energy levels (Zeeman effect)

Magnetic Field \( B_0 \)

- static magnetic field \( B_0 \)
- field strength: 1.5 – 3.0 Tesla
- homogeneity: < 1.0 ppm

- copper wires with niobium-titanium-fibers
- superconducting coil NbTi, Nb3Sn
- cooling liquid He, (N₂)
- cryostat
- nitrogen 77 K
- helium 4.2 K
- vacuum
Magnetic Field $B_0$: Construction

courtesy: Overweg, Philips

Comparison: CT - MRI

CT = transmission tomography

MRI = “direct” tomography
Correspondence Principle

in 1 mm³ water about 6x10¹⁹ protons (6x10²³ protons / mol Avogadro number)

10 ppm (10⁻⁵) energy level ratio at 1.5 T

→ 6x10¹⁴ parallel spins in M₀

Bohr’s correspondence principle

\[ \lim_{n \to \infty} QM \to \text{classical physics} \]

Proton

Summary: Proton Bulk

Quantum Mechanic ↔ Classical Mechanic

quantum mechanic

\[ \gamma B I \psi(t) = i \hbar \frac{d\psi(t)}{dt} \]

\[ \psi(t) = a(t)e^{-i\omega t/2} + b(t)e^{i\omega t/2} \]

classical mechanic

\[ \frac{dM(t)}{dt} = \gamma \cdot \vec{M} \times \vec{B} \]

notice: M is a macroscopic quantity, all nutation angles are allowed → CM
\[ \mu \] is a microscopic quantity, only +1/2 and -1/2 are allowed → QM


NMR History: Theory

Felix Bloch
- achieved the same in a sample of water
- provided the mathematical characterization of the nuclear magnetic resonance phenomenon
- Nobel Prize for physics (Bloch & Purcell) in 1952
Radiofrequency: Resonance

N Nuclear
M Magnetic
R Resonance

calculation of RF wave length:
\[ c_{\text{H}_2\text{O}} = \frac{c}{\sqrt{\varepsilon_{\text{H}_2\text{O}}}} \sim \frac{c}{7} \]
\[ \lambda = \frac{c}{v} \]
\[ \lambda \sim 67 \text{ cm at } 1.5 \text{ T (64 MHz)} \]
\[ \lambda \sim 14 \text{ cm at } 7.0 \text{ T (298 MHz)} \]

Electromagnetic Spectrum

<table>
<thead>
<tr>
<th>frequency [Hz]</th>
<th>wave length [m]</th>
<th>photon energy [eV]</th>
<th>radiation</th>
<th>molecular impact</th>
</tr>
</thead>
<tbody>
<tr>
<td>(10^6)</td>
<td>(10^{-8})</td>
<td>(10^{12})</td>
<td>x- and γ-ray</td>
<td>DNA break</td>
</tr>
<tr>
<td>(10^8)</td>
<td>(10^{-6})</td>
<td>(10^{10})</td>
<td>UV-radiation</td>
<td>visible light</td>
</tr>
<tr>
<td>(10^{10})</td>
<td>(10^{-4})</td>
<td>(10^{8})</td>
<td>e- excitation (orbital)</td>
<td>oscillation</td>
</tr>
<tr>
<td>(10^{12})</td>
<td>(10^{-2})</td>
<td>(10^{6})</td>
<td>IR-radiation</td>
<td>rotation</td>
</tr>
</tbody>
</table>

source: Lissner and Seiderer. "Klinische Kernspintomographie" 1987
Resonance: Basic Principle

- energy transfer between A and B is only possible if both systems are resonant
- RF system has to work at 64 MHz at 1.5 T

Radiofrequency: Rotating Frame

in a rotating coordinate system
the $x' - y'$ plane is rotating synchronous
with a circular polarized RF-field

→ $B_1$-vector is not moving in this system!
→ rotating $M_0$-vector only "sees" $B_1$!
Movie: Rotating Frame

Magnetization Dynamic

in the rotating frame the $M_0$-vector starts to preceed with $\omega = \gamma B_1$ around the direction of $B_1$

in the laboratory frame the $M_0$-vector is moving spirally in the direction of the $x,y$ – plane

flip angle: $\alpha = \omega t_p = \gamma B_1 t_p$

$B_0 >> B_1 \rightarrow \omega_0 >> \omega_1$
RF-Pulse Characteristics

since $t_p$ is a finite quantity the frequency distribution of the excited spins after Fourier transformation does have a frequency shape and bandwidth called \( \rightarrow \text{sinc-pulse} \)

for $t_p \rightarrow \infty$: frequency spectrum gets monochromatically

90°- and 180°- Pulses

90°-pulse ($\pi/2$-pulse) in the laboratory and rotating coordinate system

\[ N_{1/2} = N_{+1/2} \]

\[ \uparrow \rightarrow \downarrow : 3 \times 10^{14} \text{ spins per } 1 \text{ mm}^3 \]

at 1.5 T

180°-pulse ($\pi$-pulse) in the laboratory and rotating coordinate system

\[ N_{1/2} > N_{+1/2} = -M_0 \]

\[ \uparrow \rightarrow \downarrow : 6 \times 10^{14} \text{ spins per } 1 \text{ mm}^3 \]

at 1.5 T

source: Lissner and Seidner, "Klinische Kernspintomographie" 1987
• object is located in a homogeneous static magnetic field $B_0$
• RF-coil creates a magnetic field $B_1$ perpendicular to $B_0$ → transmitter
• after excitation the received signal of the object is transferred by the receiving electronic to the computer → receiver
Movie: Free Induction Decay FID

free induction decay: FID

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NMR Excitation and Signal Detection

excitation
detection
FID Signal and Frequency Spectrum

FID-signal (free induction decay) "damped oscillation"

NMR frequency spectrum

Larmor frequency $\omega_0$

Faraday Induction

bicycle
dynamo

loop with rotating magnet

rotating magnetic moments

free induction decay: FID
Signal Detection

Based on:
- Faraday law of electromagnetic induction
  and
- principal of reciprocity

Electromagnetic Induction: a temporally variable magnetic flux in a loop (receiver coil) induces a charge in this loop which is proportional to the rate of change of the magnetic flux in the loop

Principal of Reciprocity: the sensitivity for detecting a rotating magnetic moment in space is directly proportional to a corresponding electric current in the coil which is necessary for generating the same magnetic field at this point in space

Principal of Reciprocity

Hoult. Encyclopedia of Nuclear Magnetic Resonance 1996
magnetic flux through the coil:

$$\Phi(t) = \int \vec{B}_r(\vec{r}) \cdot \vec{M}(\vec{r}, t) \cdot d\vec{r}$$

Faraday induction:

$$U_{\text{ind}}(t) = -\frac{\partial \Phi(t)}{\partial t} = -\frac{\partial}{\partial t} \int \vec{B}_r(\vec{r}) \cdot \vec{M}(\vec{r}, t) \cdot d\vec{r}$$

sensitivity of receiving coil

Radio Frequency Coils: Volume Resonators
Radio Frequency Coils: Coil Arrays I

Hardy et al. MRM 2006

Radio Frequency Coils: Coil Arrays II

102 seamlessly integrated coil elements at 32 receiving channels

matrix coils:
- head
- neck
- stem
- leg

courtesy: Siemens AG, Erlangen

Zhu et al. MRM 2004
Radio Frequency Coils: Coil Sensitivity

- surface coils
- inhomogeneity correction
- phased array coils
- image combination
- parallel imaging: SMASH / SENSE

MRI Components: Schema

magnet | RF-unit (receiver)

RF-unit (transmitter) | computer

gradient system
MRI Components: Physical Parameters

- **Technical Component** → **Physical Parameter**
  - static field $B_0$ → $M_0$
  - radiofreq. RF → signal
  - gradients $G_{xyz}$ → image